Hind Foot Geometry and Proportion
By
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"The fairest thing we can experience is the mysterious. It is the fundamental emotion which stands at the cradle of true art and science. He who knows it not and can no longer wonder, no longer feel amazement, is as good as dead, a snuffed out candle" Albert Einstein (1879-1955)

## $\Phi$

"Nature holds a great mystery, zealously guarded by her custodians from those who would profane or abuse this wisdom. Periodically portions of this tradition are quietly revealed to those of humanity who have attuned their eyes to see and ears to hear. The primary requirements are openness, sensitivity, enthusiasm, and an earnestness to understand the deeper meaning of nature's marvels exhibited to us daily. Many of us tend to walk through life half asleep, at times numbed, if not actually deadened to the exquisite order that surrounds us." Scott Olsen (2009), the Golden Section: Natures Greatest Secret

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## Declaration

I hereby declare that the work within this dissertation is my own and is not the collaboration of others. Any sources used have been duly referenced.


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## Appendix

A study was carried out to see if, through the dissection of cadaver limb specimens previously trimmed to Geometric Proportions (Caldwell et al, 2010), and rigorously examined with the help of radiographs and foot mapping, it could be confirmed that the site of specified external anatomical reference points did correspond to specified internal structures. It was established through the use of digital photography and Ontrack ${ }^{\mathrm{TM}}$ software to measure very accurately the specimens which had been dissected along previously determined planes of reference. The conclusion was that yes, through the application of the above protocols, it could confidently be said that external reference points did correspond to the specified internal structures.

In the course of conducting the experiment a pattern was observed to be forming. The feet had been previously categorised according to foot shape, the greater part of these were said to be well conformed, showed good proportions, were symmetrical and balanced to their optimum proportions by the GP foot trim (Caldwell et al, 2010). When observing the saggital sections of these feet and measuring angles, a right angled triangle was seen to keep occurring. Lines were then plotted on computer from the dorsal mid-line hairline, following the angle of the dorsal wall to the point of the toe at the bearing border (dorsal hoof wall), then a line plotted from the toe, along the bearing border to the last weight bearing point of the heel (bearing border). Finally, a line plotted to join these points, from point of hairline to point of heel formed a triangle (hairline to heel), at the apex of which always formed a $90^{\text {c }}$ angle (see Figure 1).


Figure 1
Saggital section of front foot displaying Keplers Triangle

When these lines were measured it was discovered that their lengths conformed to the golden ratio. When this same procedure was applied to the other groups of feet, feet which by normal standards did not manifest good proportions or angles said to be desirable to soundness, this right angle triangle was not present.

After some research this triangle was found to be a geometric phenomenon known as "Keplers Triangle", this triangle has angles of $90^{\circ}-38.2^{\circ}-51.5^{\circ}$, (see Figure 2a). This triangle is very similar to a 3:4:5 triangle, (also known as a "pythagorean right angle"), this has angles of $90^{\circ}-36.87^{\circ}-53.13^{\circ}$. This triangle is known to obey the laws of Pythagoras:
"In any right-angled triangle the area of the square whose side is the hypotenuse, c (the side opposite the right-angle) is equal to the sum of the areas of the squares whose sides are the two legs $\mathrm{a}, \mathrm{b}$ (the two sides that meet at the right-angle)", (see Figure 2b) .

The lengths of the sides of the Kepler triangle are also known to be linked to the Golden Ratio. Livio (2002) explained "the precise value of the Golden Ratio is the neverending, never repeating number 1.6180339887 $\qquad$ ." Livio (2002) also states "In everyday life we use the word "proportion" either for the comparative relation between parts of things with respect to size or quantity or when we want to describe a harmonious relationship between different parts". As has been shown the two triangles are very similar in their angle relationships, their relative linear measurements are also very close- the $3: 4: 5$ triangle has linear length ratios of $1.00: 1.33: 1.6$ whilst the Kepler triangle linear length ratios are 1.00:1.27:1.61. These two triangles are so close in their values that they are often confused. There is a third triangle that should also be mentioned in the interest of clarity. This is known as "the Golden Triangle". This triangle is an isosceles triangle in which the two longer sides have equal lengths and in which the ratio of this length to that of the third smaller side is in the golden ratio. This triangle is also unique in that it is the only triangle to have its three angles in a $2: 2: 1$ proportion, (see Figure 2c).


## Kepler's Triangle

Angles $=90^{\circ}: 38.2^{\circ}: 51.5^{\circ}$
Linear Length Ratios
$1.00: 1.272: 1.618$

Figure 2a: Kepler's Triangle (Right Angle)


Figure 2b : 3:4:5 Triangle (Pythagorean Right Angle)


Golden Triangle
Angles $=36^{\circ}: 72^{\circ}: 72^{\circ}$
The ratio of side $a: b$ is equivalent to the golden ratio $\Phi$

Figure 2c : Golden Triangle (Isosceles)

In this study the data was compared to the Kepler's Triangle and also the golden ratio. The golden ratio is also referred to by the Greek letter phi or $\Phi$.

When the literature for foot characteristics is assessed very little can be found in text books as to a desirable model for hind foot proportions or angles. This can be attributed to the fact that at the time of writing most lameness problems involving the foot were considered to be in the front feet, this can be possibly due in part to the differing anatomical considerations and their relevant characteristics. Front feet are generally more round in shape, to cope with even weight distribution (the weight distribution is 60/40 in favour of the front feet), the horse is a flight or prey animal, (the round shape is good for rapid changes in direction. In the current Farriery text book used by all colleges, Hickman and Humphrey (1988) state: "An ideal (front) hoof is rounded at the toe and perfectly symmetrical, with the slope of the inner quarter the same as the outer quarter. The wall is thicker at the toe and gradually thinner towards the quarters, thickening again at the heels. It should be borne in mind that when a hoof is trimmed level the wall at the toe is cut at more of an angle than elsewhere which makes it appear thicker than it is. The sole is concave and the frog large and elastic with a shallow central groove and deep medial and lateral grooves. The surface of the wall is not absolutely flat but broken by a wavy growth of horn which appears as a number of rings parallel to the coronet. These rings are a normal feature and indicate alterations in the rate of growth due to either changes in food or illness. It is important to differentiate growth rings from those associated with chronic laminitis, which are widely spaced at the heels and converging at the toe. The foot axis and the angle of the wall at the heels should correspond and be between $50^{\circ}$ and $55^{\circ}$ ". This is a more in depth analysis of the shape, proportions, angles and differing parts of the front foot, whereas the hind foot description is as follows: "A hind hoof is oval at the toe and widest towards the heels. In comparison with the front hoof the sole is more concave, the frog smaller and the slope of the wall at both the inner and outer quarters more upright. The foot axis and the angle of the walls at the heels should correspond and be between $50-55^{\circ}$ as for a front."

Dollar and Wheatley (1898) state "general opinion seems to regard the best angle as somewhat less than $50^{\circ}$ in front feet and as $50^{\circ}$ or somewhat more for the hind. The greater length and obliquity of the pastern in the forelimb compensates for the greater weight and more violent shocks experienced, length and obliquity being factors
eminently more favourable to neutralising concussion. When the angle is above $55^{\circ}$ the height of the heels naturally increases in the same proportion as the length of the toe diminishes. The same conditions apply in hind feet, with exception, however that the angle formed with the earth is here somewhat greater".

Rich (1907) states that "about $53^{\circ}$ may be considered the right thing for the front foot, while $58^{\circ}-60^{\circ}$ will be the average angle for the hind foot".

Stashak (1987), states "the hind foot should present a more pointed appearance at the toe than does the forefoot. It should show evidence of breaking straight over the toe, and the frog should divide the sole into equal halves. The foot axis should be between 50 to $55^{\circ}$, and there should be no defects in the wall.

Hunting (1895) states "Looking at the foot from the side, the slope of the front should be in the same direction as the slope of the pastern. The hind foot differs from the front in being less rounded at the quarters and more pointed at the toe; it is also more upright than the fore foot and the sole much more arched. The frog is smaller partially due to the general use of caulkins".

Butler \& Butler (2004) state that "The ideal foot is symmetrical and sound. The outer surface of the wall is smooth and straight from the coronary band to the ground. The wall is angled at about $50-55^{\circ}$ at the toe in the front feet and about $2^{\circ}$ higher in the hind feet".

Price \& Fisher (1989), they say that "The hind foot be trimmed in accordance with conformation. As with the front foot the main object is to obtain the most naturally balanced foot possible".

Colles \& Ware (2010) state that "Dorso-palmer balance is assessed as for the fore-limb. It is essential that the toe is kept short, or the horse will stand with the limbs slightly forward under the body. This stance increases the load on the flexor tendons, suspensory ligaments and hocks. The fashion for shoeing using quarter-clips rather than
toe-clips is beneficial in keeping the break-over point back as far as possible, in the correct position. It is practical to shoe the hind foot with plenty of length under the heels (i.e. the shoes ending behind the most plantar bearing surface of the heels) which offers the most support to the heels.

To assess dorso-plantar balance, it is necessary to have the animal standing square, with the weight on all four feet. Viewed from the side, the hind leg should be upright with the canon (metatarsus or shannon) vertical. Viewed from behind the limb, the cannon should also be vertical, with the hock vertically above the fetlock. The hind foot is normally slightly more upright than the forefoot, usually by about $50 \%$.

Whilst this is a succinct assessment of a hind foot it is somewhat vague with little or in some cases no reference to shape, proportion, balance or symmetry. It also seemed to be in comparison with the front feet which when viewed with each of their relevant purposes borne in mind and differing forms of attachment in each limb, (the front being a purely muscular attachment at the shoulder via the scapula, whilst the hind is a complex arrangement of joints culminating in the attachment from the ab-axial to the axial skeleton via the femur/pelvic girdle).

This constant comparison between the front and hind feet does not seem to show much or any understanding of the differing anatomical or physiological considerations. As previously stated, the horse is a flight animal and the hind feet have evolved for rapid propulsion and weight bearing is a secondary design function. Smyth \& Goody (1993) state "the hind limb differs from the fore in the fact that it is directly attached, through the bony union, with the spine. This means that the propulsive forces generated by the hind limbs will be transmitted directly onto the vertebral column, but it also means that concussive forces are transmitted to it through the limb". This shows the fundamental differences between front and hind feet. It also shows that the hind foot has not really, probably by virtue of the fact that historically hind foot problems were comparatively rare, do not come under as much scrutiny as the front foot. This is now not the case with hind feet becoming more common as a cause of lameness. This may be possibly attributable to the changes in horse management and also due to modern lifestyle
changes of owners/riders with the more prevalent use of the ménage or sand school. The surfaces of such are well designed with the needs of joints and front feet in mind. They are made to give support to the front feet but also to 'give' under load, this prevents the concussive forces from being transmitted from the feet and consequently to the limb above. This use and choice of surface may be a factor in the rise in hind-limb lameness; most surfaces have as a base constituent sand with a wide range of other materials combined from worn car tyre rubber to felt. Many hind feet are now seen to be displaying what has been loosely termed 'arena fever'. The dorsal aspect of the hind foot, often worn away above and behind the harder, more abrasive resistant steel horseshoe and 'thinning' that which nature intended to be the thickest and strongest structure of the hind foot to, in some cases, the thinnest. This can have the effect that the foot can displace medially and laterally and the solear arch becoming 'flatter' and less pronounced. In some severe cases the distal border of the distal phalanx can be seen as a semi-circular bruise in the sole. The foot dimensions become wider than they are long, completely the reverse of what has been considered 'normal and healthy'. This factor also allied to the compressive nature of ménage surfaces also robs the hind foot of some of its traction in that it gives way under load.

Table A
Proposed Fore and Hind Hoof Wall Angles from Various Authors

| Author | Front Angle | Hind Angle |
| :--- | :---: | :---: |
| Hickman \& Humphrey (1988) | $50^{\circ}-55^{\circ}$ | $50^{\circ}-55^{\circ}$ |
| Dollar \& Wheatley (1898) | $<50^{\circ}$ | $>50^{\circ}$ |
| Rich (1907) | $53^{\circ}$ | $58^{\circ}-60^{\circ}$ |
| Stashak (1987) | $47^{\circ}$ | $50^{\circ}-55^{\circ}$ |
| Hunting (1895) | - | - |
| Butler \& Butler (1974) | $50^{\circ}-55^{\circ}$ | $52^{\circ}-57^{\circ}$ |
| Price \& Fisher (1989) | - | - |
| Colles \& Ware (2010) | As per conformation | $2^{\circ}$ higher than front |

### 1.1 The Golden Ratio $\boldsymbol{\Phi}$ (Phi)

The first definition of the Golden Ratio was from Euclid (325-265 BC), who defined it as "the division of a line in extreme and mean ratio". The earliest known treatise on the subject is The Divina Proportione by Luca Pacioli (1445-1517) "a monk drunk on beauty". Leonardo Da Vinci is thought to have coined the phrase Sectio Aurea or Golden Section. The first documented published use of the phrase is in Martin Ohms (1835) Pure Elementary Mathematics.

There are many names for this mathematical phenomenon: Golden or divine ratio or cut, proportion, mean, number or section are all used. The mathematical symbol is $\tau$ (tau) meaning "the cut", but now more commonly $\Phi(1.618033)$ or $\varphi(0.618033)$ (phi) is used after Phidias, who used it in his building of the Parthenon in ancient Greece, (see Figure 4 overleaf).

A complementary view of the Golden Ratio is provided by the sequence of numbers called "The Fibonacci sequence". Each number is the sum of the two previous numbers i.e. 01123581321345589 and so on ad infinitum.

The sequence connects the Golden Ratio because the ratio of the adjacent numbers in a Fibonacci series get closer and closer to the golden ratio as the numbers get bigger e.g $89 \div 55=1.618$. In nature there are many examples of the Fibonacci sequence being present, a sunflower has in common with pine cones, chrysanthemums and dahlias a double set of unequal spirals clearly evident, the pairs are always adjacent numbers in the Fibonacci series.
"Ratio (logos) is the relation of one number to another, for instance $4: 8$ ("4 is to 8 "). However proportion is a repeating ratio, this typically involves four terms, so $4: 8$ :: 5:10 (" 4 is to 8 as 5 is to 10 "). Pythagoras called this a four termed discontinuous proportion. The invariant ratio is $1: 2$, repeated in both $4: 8$ and $5: 10$. An inverted ratio reverses the terms, so $8: 4$ is the inverse of $4: 8$; the invariant ratio is now $2: 1$.

In between the two-termed ratio and the four-termed proportion is the three- termed mean. The middle term is in the same ratio to the first as it is to the last. The geometric mean between two numbers is equal to the square root of their product. So, the geometric mean of say, 1 and 9 is $\sqrt{ }(1 \times 9)=3$. This geometric mean relationship is written 1:3:9, or inverted 9:3:1. It can be written more fully as a continuous geometric proportion where these two ratios repeat the same invariant of $1: 3$, so $1: 3:: 3: 9$. The 3 is the geometric mean held in common by ratios, binding or lacing them together in what Pythagoras called a "three-termed continuous geometric proportion", Scott (2009).

### 1.2 Golden Mean Gauge/Dividers

Golden dividers and Golden mean gauges have been in use in many applications by artists, architects, craftsmen, designers, engineers, photographers, musicians, sculptors, surgeons and stock analysts.

(Figure 3a)
Leonardo da Vinci's "last supper" displaying the use of the golden ratio, (The blue lines divide the painting into their mean and extreme proportions).

(Figure 3b)
The Parthenon displaying proportions derived by applying the Golden Ratio, (The blue and yellow lines define the building by dividing the fascade and columns into their mean and extreme proportions)

(Figure 3c)
Medici violin design using the golden section (The body of the violin is divided by the green, yellow and blue coloured lines according to the golden ratio)

Golden mean dividers have already been proposed to have a use in Farriery. American farrier Craig Trnka gave a demonstration of their use in hoof proportions at the Handmade Shoes (UK) clinic in 2010. He explained "The use of a contour gauge (a comb like instrument used to measure the profile of shapes)- it is preferential to see mirror images within the foot and that is your aim despite not always achieving it, and to see a 'Golden means'- correctly proportions elements within the foot. The Golden
means is a mathematical equation that Mother Nature exists upon-ultimately achieving balance and proportion. This is a ratio of 1:1.618-your own 'golden ratio' measures can be made by having two pieces of bar stock, $131 / 8$ inches in length and riveted 5 inches from one end. The frog is shown to be the same length as the dorsal aspect-it doesn't change", Trnka (2010).

This article shows how the use of golden mean dividers may be used to proportion a hoof in the live horse. The use of the dividers shows the correct proportions of the frog length to the length of the dorsal wall, it does not however give the relationship of the length of the bearing border to the length of the frog (the linear length of the bearing border is divided into its mean and extreme ratio by the length of the true/trimmed frog). This may be due to the way in which the dividers work, the relevant lengths are at the opposite ends of the instrument and have to be reversed in order to take a reading. If the author had employed the use of a golden mean gauge the relationship in the linear lengths would have been clearly visible, the gauge has three points of measurement all in line (dividing a given length into its mean and extreme ratio) and the relationship would have been clear.

In the science of human dentistry the Golden mean has a proposed use in making Prosthodontics. According to Levin (1969) it was noted with the use of a golden means gauge, that naturally aesthetically pleasing teeth do display the golden proportion in themselves and in relation to each other, "the width of the lateral incisor to the width of the canine is also in the golden proportion, as is the width of the canine to the first premolar. The widths of the incisors are in the golden proportion to each other as seen from the front". He has also noted that individual teeth display the golden ratio in their own proportions, "the contact point or interdental papillae tip divided the length of the clinical crown (or interdental space) into the golden proportion", (Levin (1969)).

Thus, the golden mean has an accepted use in many scientific, artistic, architectural, musical and engineering applications (see Figures 3a to 3c above) where the exponents have opted to mimic the harmonious proportions this mathematical phenomenon of nature provides.

This thesis aims to investigate the phenomenon in hind feet of horses.

## 2 Aim

The aim of this study was to investigate whether, in a sample of 35 randomly selected cadaver hind feet from the abattoir that had been trimmed to Geometric Proportions according to Caldwell et al (2010):
a) The Geometric Progression ( $\Phi$ ) existed in the following measurements of:
(1) Dorsal wall length
(2) Bearing border length
(3)Hairline to heel length
b) Clear indication of a defined dorsal wall hind hoof angle can be given; and
c) A $90^{\circ}$ angle exists at the hairline to heel angle.


Materials

1. 35 cadaver limb specimens chosen at random (provided by Mr H Hawkins \& Sons).
2. Erbauer ERB 2501SE 255 mm sliding mitre saw (saw blade 54 TPI).
3. Freezer.
4. Leg vice.
5. Nikon D 3000 Digital Camera ( $18-55 \mathrm{~mm}$ lens)
6. Neewer 50" Camera Tripod.
7. Laptop (hp pavilion dv6).
8. Ontrack ${ }^{\text {TM }}$ software.
9. Photographic box with steel rule attached for calibration.
10. Clean, dry cloth.
11. Latex gloves.
12. Hibi scrub.
13. Dry line marker.
14. Toothbrush.
15. Soft nylon brush.
16. Coveralls.
17. Particle masks.
18. Specimen holding jig.
19. Camping Gas stove.
20. Steel water container (see Figure 2)
21. Black \& Decker workstation.
22. Ratchet Cramps (x 2)
23. Safety Glasses.
24. Clean hand towel.
25. Irwin jack universal saw (22")
26. Hoof Gauge.
27. 13mm Ratchet Spanner.
28. Gas Lighter.
29. Notebook \& pen.
(See Figure 5).


Figure 5
A selection of materials used in the study

## 4

Methods
The feet were supplied by the abattoir having been severed at the metatarsal/phalangeal articulation in a frozen state. This was done for two reasons, firstly to prevent decomposition and secondly to keep the soft tissue in a stable state which made measurement easier. The feet were then mapped out and trimmed according to the Geometric Proportions trim method given by Caldwell et al (2010) (see Figure 6a).


Figure 6a
Foot mapping for GP trimming protocol

The phalanges were kept attached at this stage as they provided a useful aid to grip the upturned feet in the vice whilst trimming. Once the feet had been trimmed the pastern was then removed (see Figure 6b).


Figure 6b
Solear view of hind foot prepared to the trimming protocol

A transverse section of the foot was taken just proximal to the hairline, at approximately $90^{\circ}$ to the bone column. This was done to make it easier to subsequently take a saggital section of the hoof capsule. The foot was then placed in the vice of the saw. In the previous studies feet had been dissected in a band-saw. This introduced possible human errors as the cutting can only be done free-hand. After some research a Sliding mitre saw was identified as the best tool to accommodate the task as it has a sliding rail which kept the blade at $90^{\circ}$ to the vice. A small "holding" jig was manufactured to hold the feet in place whilst being cut. (See Figure 7).


Figure 7
Foot Holding Jig

The jig consisted of two pieces of angle iron, $75 \mathrm{~mm} \times 75 \mathrm{~mm} \times 6 \mathrm{~mm}, 150 \mathrm{~mm}$ long. These had three holes drilled and tapped to 10 mm metric thread pattern, they were drilled in a diagonal line to best accommodate varying foot sizes. Four 10 mm bolts, 75 mm long had a blunt point ground on the end to grip the hoof wall whilst cutting. The angle then had a piece of $25 \mathrm{~mm} \times 12 \mathrm{~mm}$ flat bar, 150 mm long welded inside the web of the angle iron, thus enabling the jig to be cramped to the saw-bed, (see Figure 7).

This eliminated any human errors of judgement that can be made whilst 'free cutting' with a band saw and is a much safer procedure. The saw blade has tungsten carbide teeth; this gives a much cleaner, smoother surface to be cut and subsequently photographed. The saw also has a laser marker precisely following the path of the cut, this made positioning the foot at the mid-line dead centre of the dorsal wall (after Reilly $(1996,98)$ ) to be sectioned much more accurate. (See Figure 8).


Figure 8
Arrangement for cutting feet showing foot holding jig in place

Once the foot has been sectioned it was cleaned using a nylon bristled brush. This was used to remove the frozen hoof swarf. It was then cleaned on the internal saggital section surface using a clean cloth and clean hot water (this prevented the water rapidly freezing in cold weather and creating a highly reflective sheen on the surface to be photographed).


Figure 9
35 Sagitally sectioned Cadaver hind feet

The medial side of the hoof was then placed in the calibration box. This had a steel rule fixed to the front to give an aid to scale and to allow calibration prior to analysis on Ontrac ${ }^{\mathrm{TM}}$ once the photographs had been loaded on to this system. The camera was set up on the tripod and set at a measured distance; 860 mm from the lens to the back of the photographic box. The feet were numbered and then photographed (the camera was set on auto-focus) (See Figure 9).


Figure 10a
Lateral side view of hind foot to identify last point of heel and show other points of measurement.

The feet were photographed from the lateral side first. It was found in the previous practical analysis study (in the $2^{\text {nd }}$ year of the Foundation Degree at Myerscough) that once sectioned at the mid-line dead centre of the hoof wall, the last point of weight bearing heel was not visible from the internal photograph. As this was one of our points of measurement in order to measure bearing border length it was necessary to establish this first (see Figure 10a). Once the lateral shots had been taken then the section was rotated by $180^{\circ}$ and the internal saggital section was photographed in turn.

The photographs were then downloaded from the digital camera onto the lap-top. These were then loaded on to Ontrac ${ }^{\mathrm{TM}}$ and were analysed.

Lines were then plotted on to the photograph (once the system has been calibrated to ensure accuracy and continuity). These lines were as follows:

1. Line from dorsal midline hairline, following the angle of the dorsal wall to the point of the toe at the bearing border (termed dorsal hoof wall) (see Figure 8b);
2. Line plotted from the toe, along the bearing border to the last weight bearing point of the heel (termed bearing border, see Figure 10b); and a
3. Line plotted to join these points, from point of hairline to last weight bearing point of the heel (termed hairline to heel) (See Figure 10b).


Figure 10b
Saggital section showing measurement parameters.

These lines, once plotted were then measured individually and the angles they made with each other measured and recorded (see Figure 10c). This was the basis of the raw data for analysis.


Figure 10c
Saggital section showing angles to be measured.

Thus, the 5 parameters to be measured were:

| Lengths: | Dorsal Hoof Wall Length <br> Bearing Border Length <br> Hairline to Heel Length |
| :--- | :--- |
| Angles: | Dorsal Wall Angle <br> Hairline to Heel Angle |

## 5 Primary Results

### 5.1 Table 1 - Lateral Side Measurements (External)



### 5.2 Results - Descriptive Statistics for Lateral Side



## Descriptive Statistics: Bearing Border Length (mm)

Total

| Variable | Count | N N* | Mean | SE Mean | StDev | Variance |  |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| Bearing Border Length (m | 35 | 35 | 0 | 104.15 | 1.69 | 9.99 | 99.77 |

N for

| Variable | Sum Minimum | Median | Maximum Range | Mode | Mode |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Bearing Border Length (m | 3645.20 |  | 78.82 | 105.26 | 122.12 | 43.30 | $* 0$ |
| Variable | Skewness | Kurtosis |  |  |  |  |  |
| Bearing Border Length (m | -0.51 | -0.18 |  |  |  |  |  |

Descriptive Statistics: Hairline to Heel Length (mm)

|  | Total |  |  |  |  |  |  |
| :--- | :---: | :---: | :---: | ---: | ---: | ---: | ---: |
| Variable | Count | N N* | Mean | SE Mean | StDev | Variance |  |
| Hairline to Heel Length | 35 | 35 | 0 | 82.35 | 1.26 | 7.47 | 55.74 |

N for

| Variable | Sum Minimum | Median Maximum | Range | Mode | Mode |  |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: |
| Hairline to Heel Length | 2882.13 | 65.72 | 84.15 | 95.05 | 29.33 | $* 0$ |


| Variable | Skewness | Kurtosis |
| :--- | ---: | ---: |
| Hairline to Heel Length | -0.50 | -0.55 |



### 5.3 Table 2 - Saggital Side Measurements (Internal)

|  | Dorsal Hoof <br> Wall <br> Length <br> $(\mathrm{mm})$ | Bearing <br> Border <br> Length $(\mathrm{mm})$ | Hairline to <br> Heel <br> Length <br> $(\mathrm{mm})$ | Dorsal <br> Wall <br> Angle $\left({ }^{\circ}\right)$ | Hairline to <br> Heel <br> Angle $\left({ }^{\circ}\right)$ |
| :--- | :---: | :---: | :---: | :---: | :---: |
|  | 79.94 | 121.93 | 99.47 | 54.4 | 85.5 |
| Foot 1a | 103.65 | 84.18 | 54.5 | 85.8 |  |
| Foot 2a | 65.84 | 114.36 | 88.79 | 50.88 | 88.7 |
| Foot 3a | 76.66 | 99.37 | 80.1 | 53 | 85.6 |
| Foot 4a | 65.21 | 102.68 | 76.88 | 48.4 | 86.6 |
| Foot 5a | 72.94 | 86.74 | 69.71 | 51.9 | 84.7 |
| Foot 6a | 59.03 | 96.01 | 83.86 | 56.9 | 74.6 |
| Foot 7a | 74.52 | 92.05 | 77.65 | 55.5 | 80.8 |
| Foot 8a | 65.02 | 92.92 | 68.2 | 47.4 | 91.2 |
| Foot 9a | 61.42 | 91.22 | 75.96 | 55.8 | 83.8 |
| Foot 10a | 60.17 | 128.73 | 98.33 | 49.8 | 90.3 |
| Foot 11a | 82.73 | 102.81 | 80.02 | 50.03 | 83.3 |
| Foot 12a | 74.78 | 96.76 | 74.98 | 50.5 | 87.7 |
| Foot 13a | 65.64 | 116.61 | 92.43 | 53.9 | 84.7 |
| Foot 14a | 73.96 | 89.33 | 72.34 | 53.4 | 84.5 |
| Foot 15a | 60.35 | 90.94 | 73.14 | 53.5 | 84.9 |
| Foot 16a | 60.01 | 113.58 | 90.46 | 52.5 | 85.7 |
| Foot 17a | 76.58 | 84.64 | 69.79 | 55 | 80.1 |
| Foot 18a | 60.92 | 102.72 | 80.13 | 50.5 | 84.3 |
| Foot 19a | 72.55 | 100.62 | 78.45 | 51 | 85.4 |
| Foot 20a | 69.98 | 88.14 | 70.04 | 52 | 82.4 |
| Foot 21a | 63.91 | 103.22 | 77.87 | 48.7 | 83.3 |
| Foot 22a | 77.53 | 110.6 | 82.1 | 47.5 | 91 |
| Foot 23a | 72.97 | 109.02 | 80.52 | 47.7 | 89.6 |
| Foot 24a | 73.9 | 102.77 | 74.79 | 46.7 | 94.8 |
| Foot 25a | 64.32 | 13.7 |  |  |  |
| Foot 26a | 78.34 | 116.88 | 89.64 | 49.7 | 87.9 |
| Foot 27a | 83.23 | 116.06 | 85.93 | 47.7 | 87 |
| Foot 28a | 76.12 | 116.07 | 95.09 | 54.5 | 84.5 |
| Foot 29a | 72.26 | 114.84 | 89.87 | 51.6 | 89.6 |
| Foot 30a | 83.42 | 123.06 | 91.52 | 48.2 | 89 |
| Foot 31a | 76.19 | 107.88 | 88.07 | 53.9 | 81.8 |
| Foot 32a | 82.97 | 130 | 100.94 | 50.8 | 89.4 |
| Foot 33a | 69.89 | 110.13 | 81.17 | 47.4 | 93.3 |
| Foot 34a | 66.23 | 95.15 | 69.01 | 46.3 | 89.8 |
| Foot 35a | 69.37 | 116.25 | 92.68 | 52.5 | 90.4 |
|  |  |  |  |  |  |

### 5.4 Descriptive Statistics for Saggital Section

## Descriptive Statistics Dorsal Hoof Wall Length (mm)



## Descriptive Statistics: Bearing Border Length (mm)

|  | Total |  |  |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | ---: |
| Variable | Count | N | $\mathrm{N}^{*}$ | Mean | SE Mean | StDev | Variance |  |
| Bearing Border Length $(\mathrm{m}$ | $\cdot 35$ | 35 | 0 | 105.36 | 2.07 | 12.23 | 149.55 |  |

N for

| Variable | Sum Minimum Median Maximum Range Mode Mode |  |  |  |  |  |  |
| :--- | ---: | ---: | :--- | :--- | :--- | :--- | :---: | :---: |
| Bearing Border Length (m | 3687.74 | 84.64 | 103.22 | 130.00 | 45.36 | $*$ | 0 |

## Descriptive Statistics: Hairline to Heel Length (mm)

| Total |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Variable | Count | $\mathrm{N} \mathrm{N}^{*}$ | Mean | SE Mean | StDev | Variance |
| Hairline to Heel Length | 35 | 350 | 82.40 | 1.56 | 9.25 | 85.55 |
| N for |  |  |  |  |  |  |
| Variable | Sum Minimum Median |  |  | Maximum | Range | de Mode |
| Hairline to Heel Length | 2884.1 | 68.20 | 80.52 | 100.94 | 32.74 | * 0 |

## Descriptive Statistics: Dorsal Wall Angle ( ${ }^{\circ}$ )

|  | Total |  |  |  |  |  |  |
| :--- | :---: | :---: | :---: | ---: | ---: | ---: | :---: |
| Variable | Count | N | $N^{*}$ | Mean | SE Mean | StDev | Variance |
| Dorsal Wall Angle $\left({ }^{\circ}\right)$ | 35 | 35 | 0 | 51.257 | 0.496 | 2.933 | 8.600 |


| Variable | Sum | Minimum | Median | Maximum | Range |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Dorsal Wall Angle ( ${ }^{\circ}$ ) | 1794.010 | 46.300 | 51.000 | 56.900 | 10.600 |


|  | N for |  |  |  |
| :--- | :---: | :---: | ---: | ---: |
| Variable | Mode | Mode | Skewness | Kurtosis |
| Dorsal Wall Angle $\left({ }^{\circ}\right) 47.4,47.7,50.5,52.5$ | 2 | 0.02 | -1.08 |  |

The data contain at least five mode values. Only the smallest four are shown.

## Descriptive Statistics: Hairline to Heel Angle ( ${ }^{\circ}$ )

| Total |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Variable <br> Hairline to Heel Angle | Count N N* |  | * Mean | SE Mean | StDev | Variance |  |
|  | 3535 | 0 | 86.343 | 0.678 | 4.012 | 16.0 |  |
| Variable | Sum |  | inimum | Median | Max | imum | Range |
| Hairline to Heel Angle ( | ( 3022.000 |  | 74.600 | 85.700 |  | 4.800 | 20.200 |

The data from the previous tables was then analysed using the Minitab $15^{\mathrm{TM}}$ statistical analysis program. Data was considered in the following way: the "primary data" was the raw unadulterated data and the "secondary data" refers to that which has been converted to for example, ratios.

### 5.5 Primary Data Results - Lateral Side Descriptive Statistics

The dorsal hoof wall length mean was $70.65 \mathrm{~mm} \pm 6.40 \mathrm{~mm}$ standard deviation, with a standard error mean of 1.08 mm .

The minimum length was 56.86 mm and the maximum length was 86.48 mm .
This data set had a range of 86.48 mm and had a variance of 40.93 mm from the mean value.

The mode value was 29.62 mm

The bearing border length mean was $104.15 \mathrm{~mm} \pm 9.99 \mathrm{~mm}$ standard deviation, with a standard error mean of 1.69 mm . The minimum length was 78.2 mm and the maximum length was 122.12 mm .

This data set had a range of 122.13 mm and had a variance of 99.77 mm from the mean value.

The mode value was 43.30 mm .

The hairline to heel length mean was $82.35 \mathrm{~mm} \pm 7.47 \mathrm{~mm}$ standard deviation, with a standard error mean of 1.26 mm .

The minimum length was 65.72 mm and the maximum length was 95.05 mm .
This data set had a range of 95.05 mm and had a variance of 55.74 mm from the mean value.

The mode value was 29.33 mm .

The dorsal hoof wall angle mean was $52.046^{\circ} \pm 2.736^{\circ}$ standard deviation, with a standard error mean of $0.463^{\circ}$.

The minimum angle was $47.5^{\circ}$ and the maximum angle was $58.7^{\circ}$.
This data set had a range of $11.200^{\circ}$ and had a variance or $7.487^{\circ}$ from the mean value.
The mode value was $54.1^{\circ}$

The hairline to heel angle mean was $85.481^{\circ} \pm 4.181^{\circ}$ standard deviation, with a standard error mean of $0.707^{\circ}$.
The minimum angle was $76.7^{\circ}$ and the maximum was $92.4^{\circ}$.
This data set had a range of $92.400^{\circ}$ and had a variance of $17.480^{\circ}$ from the mean value. There were 4 mode values in this data set: $81.9^{\circ}, 82.2^{\circ}, 85.5^{\circ}, 86.9^{\circ}$.

### 5.6 Primary Data Results - Saggital Side Descriptive Statistics

The dorsal hoof wall length mean was $71.11 \mathrm{~mm} \pm 7.41 \mathrm{~mm}$ standard deviation with a standard error mean of 1.25 mm .

The minimum length was 59.03 mm and the maximum was 83.42 mm .
This data set had a range of 24.39 mm and had a variance of 54.94 mm from the mean value.

The bearing border length mean was $105.66 \mathrm{~mm} \pm 12.23 \mathrm{~mm}$ standard deviation with a standard error mean of 2.07 mm . The minimum length was 84.64 mm and the maximum was 128.73 mm .

This data set had a range of 45.36 mm and a variance of 149.55 mm from the mean value.

The hairline to heel length mean was $82.40 \mathrm{~mm} \pm 9.25 \mathrm{~mm}$ standard deviation with a standard error mean of 1.56 mm .

The minimum length was 68.2 mm and the maximum was 99.47 mm .
This data set had a range of 32.74 mm and a variance of 85.55 mm from the mean value.

The dorsal wall angle mean was $51.257^{\circ} \pm 2.933^{\circ}$ standard deviation with a standard error mean of $0.496^{\circ}$.

The minimum angle was $46.3^{\circ}$ and the maximum was $55.8^{\circ}$
This data set had a range of $10.600^{\circ}$ and a variance of $8.600^{\circ}$ from the mean value.

The hairline to heel angle mean was $86.343^{\circ} \pm 4.012^{\circ}$ standard deviation with a standard error mean of $0.678^{\circ}$.

The minimum angle was $74.6^{\circ}$ and the maximum was $94.8^{\circ}$.
This data set had a range of $20.2^{\circ}$ and a variance of $16.097^{\circ}$.

### 5.7 Discussion of Primary Results

When the primary results of the data tables are viewed they are showing the true biological spread of the data. This shows how all of the cadaver feet are made up with all of their differing physical sizes, shapes and proportions on display. They show how diverse the population sample was. The reason the lateral side photographs were taken at the beginning of the experiment was to establish the true point of the heel in the trimmed foot as this was not visible from the saggital section. This datum point was necessary to create a triangle and measure the true linear lengths and angles in the saggital section photographs. The differences between the measurements for the lateral side and the saggital section are as follows:

| Dorsal hoof wall length | $\pm 1.12 \mathrm{~mm}$ |
| :--- | :--- |
| Bearing border length | $\pm 1.51 \mathrm{~mm}$ |
| Hairline to heel length | $\pm 0.05 \mathrm{~mm}$ |
| Dorsal wall angle | $\pm 0.8^{\circ}$ |
| Hairline to heel angle | $\pm 0.9^{\circ}$ |

The above results show how accurate the data collection was using the Ontrac ${ }^{\text {TM }}$ software program. The relatively small difference in the hairline to heel measurement may be due to the fact that the dorsal wall length was the first measurement taken, followed by the bearing border length. The hairline to heel length was then taken to create a triangle, in other words the hairline to heel length was measured last, between two reference points (at the dorsal hairline and the caudal heel) that were created by doing the other measurements first ie.; it is not an external measurement that is normally visable.

The linear lengths of both data sets were then individually transformed into ratios. This was necessary because although the primary data was more statistically powerful this had to be changed to enable a direct comparison with the golden ratio. The hypothesis
being that if the dorsal wall length was divided by itself this would give the ratio of 1.00, the hairline to heel length would equate to a ratio of 1.272 and the bearing border length would equate to a ratio of 1.618 (the golden ratio) and so create a 'Kepler triangle' with angles of $90^{\circ}$ (hairline to heel) and $51.5^{\circ}$ (Dorsal wall angle).

The ratios are calculated by taking the shortest linear length (this becomes ratio 1.00 ) and dividing the other linear lengths by this figure. These become the second and third values in the Kepler triangle equation.

The results of this calculation are displayed in the table of results overleaf. (see Table C).

The dorsal wall angle for the lateral side mean was $52.046^{\circ} \pm 2.736^{\circ}$.
The dorsal wall angle for the saggital sections was $51.257^{\circ} \pm 2.933^{\circ}$. This is compared to an ideal angle of $51.5^{\circ}$ for a kepler triangle. When the mean calculated for the two data sets it was found to be $51.7^{\circ}, 0.2^{\circ}$ away from the golden ratio $\left(51.5^{\circ}\right)$.

The hairline to heel angle mean for the lateral side was $85.481^{\circ} \pm 4.181^{\circ}$. The hairline to heel angle mean for the saggital section was $86.343^{\circ} \pm 4.012^{\circ}$. When the mean was calculated for the two data sets it was found to be $85.9^{\circ}, 4.1^{\circ}$ away from the golden ratio $\left(90^{\circ}\right)$.

## Table B

Comparison of Angle Results against Kepler Triangle Angles

| Angles | Lateral | Saggital | Kepler |
| :--- | :--- | :--- | :--- |
| Dorsal wall angle mean | $52.046^{\circ}$ | $51.257^{\circ}$ | $51.5^{\circ}$ |
| Hairline to heel angle mean | $85.481^{\circ}$ | $86.343^{\circ}$ | $90^{\circ}$ |

Table C
Lateral \& Saggital Ratios

|  | Lateral Side Ratios |  |  |  | Saggital Side Ratios |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Ratio | Ratio | Ratio |  | Ratio | Ratio | Ratio |
|  | B/B | C/B | D/B |  | B/B | C/B | D/B |
| Foot 1 | 1.00 | 1.56 | 1.24 | Foot 1a | 1.00 | 1.53 | 1.24 |
| Foot 2 | 1.00 | 1.46 | 1.27 | Foot 2a | 1.00 | 1.57 | 1.28 |
| Foot 3 | 1.00 | 1.54 | 1.21 | Foot 3a | 1.00 | 1.49 | 1.16 |
| Foot 4 | 1.00 | 1.43 | 1.17 | Foot 4a | 1.00 | 1.52 | 1.23 |
| Foot 5 | 1.00 | 1.43 | 1.08 | Foot 5a | 1.00 | 1.41 | 1.05 |
| Foot 6 | 1.00 | 1.40 | 1.11 | Foot 6a | 1.00 | 1.47 | 1.18 |
| Foot 7 | 1.00 | 1.33 | 1.13 | Foot 7a | 1.00 | 1.29 | 1.13 |
| Foot 8 | 1.00 | 1.37 | 1.10 | Foot 8a | 1.00 | 1.42 | 1.19 |
| Foot 9 | 1.00 | 1.47 | 1.13 | Foot 9a | 1.00 | 1.51 | 1.11 |
| Foot 10 | 1.00 | 1.49 | 1.28 | Foot 10a | 1.00 | 1.52 | 1.26 |
| Foot 11 | 1.00 | 1.56 | 1.26 | Foot 11a | 1.00 | 1.56 | 1.19 |
| Foot 12 | 1.00 | 1.43 | 1.12 | Foot 12a | 1.00 | 1.37 | 1.07 |
| Foot 13 | 1.00 | 1.48 | 1.15 | Foot 13a | 1.00 | 1.47 | 1.14 |
| Foot 14 | 1.00 | 1.51 | 1.22 | Foot 14a | 1.00 | 1.58 | 1.25 |
| Foot 15 | 1.00 | 1.45 | 1.18 | Foot 15a | 1.00 | 1.48 | 1.20 |
| Foot 16 | 1.00 | 1.52 | 1.23 | Foot 16a | 1.00 | 1.52 | 1.22 |
| Foot 17 | 1.00 | 1.58 | 1.22 | Foot 17a | 1.00 | 1.48 | 1.18 |
| Foot 18 | 1.00 | 1.39 | 1.16 | Foot 18a | 1.00 | 1.39 | 1.15 |
| Foot 19 | 1.00 | 1.37 | 1.09 | Foot 19a | 1.00 | 1.42 | 1.10 |
| Foot 20 | 1.00 | 1.37 | 1.09 | Foot 20a | 1.00 | 1.44 | 1.12 |
| Foot 21 | 1.00 | 1.40 | 1.15 | Foot 21a | 1.00 | 1.38 | 1.10 |
| Foot 22 | 1.00 | 1.29 | 0.99 | Foot 22a | 1.00 | 1.33 | 1.00 |
| Foot 23 | 1.00 | 1.58 | 1.21 | Foot 23a | 1.00 | 1.52 | 1.13 |
| Foot 24 | 1.00 | 1.52 | 1.18 | Foot 24a | 1.00 | 1.48 | 1.09 |
| Foot 25 | 1.00 | 1.54 | 1.15 | Foot 25a | 1.00 | 1.60 | 1.16 |
| Foot 26 | 1.00 | 1.49 | 1.16 | Foot 26a | 1.00 | 1.49 | 1.14 |
| Foot 27 | 1.00 | 1.29 | 0.96 | Foot 27a | 1.00 | 1.39 | 1.03 |
| Foot 28 | 1.00 | 1.55 | 1.27 | Foot 28a | 1.00 | 1.52 | 1.25 |
| Foot 29 | 1.00 | 1.59 | 1.25 | Foot 29a | 1.00 | 1.59 | 1.24 |
| Foot 30 | 1.00 | 1.47 | 1.07 | Foot 30a | 1.00 | 1.48 | 1.10 |
| Foot 31 | 1.00 | 1.44 | 1.18 | Foot 31a | 1.00 | 1.42 | 1.16 |
| Foot 32 | 1.00 | 1.64 | 1.26 | Foot 32a | 1.00 | 1.57 | 1.22 |
| Foot 33 | 1.00 | 1.56 | 1.15 | Foot 33a | 1.00 | 1.58 | 1.16 |
| Foot 34 | 1.00 | 1.45 | 1.07 | Foot 34a | 1.00 | 1.44 | 1.04 |
| Foot 35 | 1.00 | 1.77 | 1.41 | Foot 35a | 1.00 | 1.68 | 1.34 |
| Mean | 1.00 | 1.48 | 1.17 | Mean | 1.00 | 1.48 | 1.16 |

### 5.8 Secondary Results

After both sets of data were transformed into ratios and the mean was found in each linear length it could be seen that they were nearly identical. This allowed the data to be analysed as one, as what was said of the lateral side measurements would be the same for the saggital sections.


The above scatter-plot shows both the lateral side measurements and the saggital side measurements. Each dot represents a foot and shows how close both of the data sets are. If the feet were all trimmed in the golden ratio then they would be on or close to the 1.6 mark. The groupings show that although they are not all exactly 1.6 they reasonably close and in a similar range.


This graph shows the spread of the ratios for the lateral side. It shows that the majority of feet (26) are in the $1.40-1.59$ ratio range.


This graph shows the spread of the ratios for the saggital section. It shows that the majority of feet (27) are in the $1.40-1.59$ ratio range.

The above scatter-plot and graphs show the relationship of the two data sets to each other. They show that the data is close enough to be considered as the same.

### 5.9 Discussion of Secondary Results

The data from the secondary results shows clearly that the Lateral side measurements results and the saggital section measurement results are almost identical. Any differences at this level may easily be accounted for by human error and the level of accuracy in marking up the photographs with the Ontrack ${ }^{\mathrm{TM}}$ program. When the photographs were loaded on to Ontack ${ }^{\mathrm{TM}}$ the screen size would have adequately displayed a much larger photograph, any future work may find it beneficial to increase the initial magnification of the photograph. This will enable a much more accurate positioning of the relevant datum points. The choice of computer measuring software may also be worthy of further exploration as there are other software programs available (Metron?)

The initial selection of the cadaver limb specimens may also be an area for further changes. Many of the feet supplied had been shod or trimmed relatively recently pre euthanasia and the amount of hoof horn growth was minimal to the extent that it did lead to some compromises in the trimming protocol, some of the feet were possibly over-trimmed by the previous farrier/s in the region of the toe of the foot. The sole was over-dressed and consequently the dorsal hoof wall length was thinned in this area, when the measurements were taken some of the lines of reference had to be estimated. This was done by taking the proximal third of the dorsal hoof wall as the true hoof angle, Curtis (1999). Almost all of the feet showed that the dorsal wall had been 'dumped', the wall when viewed laterally did not display a straight line from hairline to distal toe/bearing border. This is almost certainly due to the universal application of quarter-clipped hind shoes, the toe of the foot is fitted protruding through the front of the shoe and 'rounded' as a safety precaution against interference with the front limbs. A point of note was that all of the feet needed to be trimmed at the heels. With a selection of cadaver limb specimens that had more foot growth present would enable a much more accurate adherence to a strict trimming protocol and consequently a more accurate primary data set for comparison.

The relatively small sample size of the data set is also an area for expansion. The 35 feet used in this study mean that the data collected does have limited statistical value. To enable more powerful statistical conclusions to be drawn from the data sets in this study a paired t-test could have been performed; this would have given the statistical results more value. In a future study a larger sample size would be statistically advantageous, a sample population of 100 cadaver hind feet would be numerically ideal.

The results of the secondary data show that with regard to the relationship to keplers triangle and the golden ratio they are very close indeed. The dorsal wall hoof angle is the closest being only $0.2^{\circ}$ away from the corresponding angle in the keplers triangle. The hairline to heel angle is next closest in relation to the keplers triangle being only $4.1^{\circ}$ away from the ideal corresponding angle in a keplers triangle.

The ratio linear lengths are the furthest from the ideal of the golden ratio lengths but even these are extremely close, the hairline to heel length mean was 1.17 against an ideal 1.27 in the golden ratio. The bearing border length mean was 1.48 against an ideal of 1.618 in the golden ratio. These equate to a difference of 0.1 and 0.138 respectively.

These minute differences may in real terms be a single pass of the file side of a farriers rasp.

In the data sets that have been analysed the foot that is closest in comparison to the golden ratio was foot 13/13a:
(see Figure 12)


Figure 12
Foot Number 13

Dorsal hoof wall length mean 68.5 mm .
Bearing border length mean 101.01 mm .
Hairline to heel length mean 78.51 mm .
Dorsal wall angle mean $50.7^{\circ}$.
Hairline to heel angle mean $87.3^{\circ}$.

When looking at the data collected and the relative closeness of the feet trimmed and compared to the kepler triangle and its relation to the golden ratio, it may be that some small changes to the trimming protocol may enable any future work to further enhance the relationship to the golden ratio. When the data is viewed it would seem that the dorsal wall angle is almost identical at 51.7 degrees, the corresponding kepler triangle angle is 51.5 degrees. The hairline to heel angle at 85.9 degrees seems to be where the error is starting to occur, the corresponding kepler triangle angle is 90 degrees. This is then borne out further by the ratio lengths, the mean hairline to heel length ratio value was 1.17 , the corresponding kepler triangle value was 1.27 and the bearing border length mean was 1.48 , the corresponding value was 1.618 . This analysis of the results shows that for the golden ratio to be undeniably present and correct to all of the five parameters, the three linear length ratios and both of the angles measured, the bearing
border length would need to be increased. This would have the effect of increasing the ratio length (by 0.138 ) this would in turn increase the hairline to heel ratio length (by 0.1 )and will also increase the hairline to heel angle to the required 90 degrees. To achieve this it would mean a combination of not trimming the area at the toe by a negligibly small amount and trimming more at the heel area, again by a small amount. This would correctly proportion the trimmed hind hoof to the golden ratio.

## 6 Conclusion

In answer to the aims of this study which were:

To investigate whether, in a sample of 35 randomly selected cadaver hind feet from the abattoir that had been trimmed to Geometric Proportions according to Caldwell et al (2010):
a) The Geometric Progression ( $\Phi$ ) existed in the following measurements of:
(1) Dorsal wall length
(2) Bearing border length
(3) Hairline to heel length
b) Clear indication of a defined dorsal wall hind hoof angle can be given; and
c) A $90^{\circ}$ angle exists at the hairline to heel angle.

That no, the Geometric progression was not present in the saggital sections of the hind feet when this trimming protocol was adhered to. They were however extremely close, the dorsal wall angle was $0.2^{\circ}$ different.

It would seem that a clear definition of a dorsal wall angle would be between $51.5^{\circ}$ and $52^{\circ}$.

As the Geometric progression was not definitively present, the hairline to heel angle was not $90^{\circ}$.

The results that this study produced show that in order to define a kepler triangle in hind feet the linear lengths would need to extend caudally. When the feet were trimmed to
the protocol it would seem that the heels were already lowered to their finite level, in a live animal this would almost certainly be injurious. This removes any margin for extending the trim caudally.

The Golden ratio could however be advantageous in proscribing the optimum length when fitting a shoe. This extension of the linear lengths by the application of the correct size horseshoe would have the effect of creating the kepler triangle linear lengths, creating a $90^{\circ}$ at the hairline, defining a dorsal wall angle of $51.5^{\circ}$ and possibly enhancing the biomechanical functions of the foot.

Farriery science it would seem is new to the ideas and application of the Golden ratio and its possibilities to enhance biomechanical function. Other industries have grasped its concepts and have seen the benefits of its ability to apply pleasing proportions and the enhancing of form and function.

## Appendix

Histograms showing primary data from lateral side







Histogram of Bearing Border Length (mm)
Normal


| Mean | 105.4 |
| :--- | ---: |
| StDev | 12.23 |
| N | 35 |

N




All of the above statistics and histograms are of the saggital side.

## "Equipment"

Edgar A. Guest.
Figure it out for yourself my lad,
You have all the greatest have had, Two arms, two legs, two hands, two eyes,

And a brain to use if you'd be wise,
With this equipment they all began,
So start for the top and say, 'I can'.
Look them over, the wise and the great,
They take their food from a common plate,
And similar knives and forks they use,
With similar laces they tie their shoes,
The world considers them brave and smart, But you have all they had when they made their start.

You can triumph and come to skill, You can be great if you only will, You're well equipped for the fight you choose, You have arms and legs and a brain to use, And men who have risen great deeds to do, Began their life with no more than you.

You are the handicap you must face, You are the one, who must choose your place,

You must say where you want to go,
How much you'll study the truth to know,
God has equipped you for life, but he
Lets you decide who you want to be.
The courage must come from the soul within,
The man must furnish the will to win,
So figure it out for yourself my lad,
You were born with all the great have had,
With your equipment they all began,
So get hold of yourself and say 'I can'.

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