The Golden Ratio In Relation to Hoof Capsular Geometry In Front Feet

By
P.W.Balchin

"To the rational being, only the irrational is unendurable"

Epictetus (55-135 A.D)

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(tau)

Abstract

Four hoof metrics: The dorsal wall length, bearing border length and hairline to (last weight bearing point of) heel length and hairline angle of fifty pairs of randomly selected fore feet were investigated. Prior to investigation the hoof capsules had been trimmed to a strict and defined protocol. The trimming protocol was based on the premise that optimum hoof balance tailored to an individuals conformation can be obtained by attempting to align external reference points to corresponding internal structures. This investigation concentrated on dorso-palmar hoof measurement in a saggital plane, to ascertain as to whether or not the proportional lengths obtained from certain measurements taken from saggital sections correlated with the progressive geometrics known to form the Golden Ratio.

From saggital sections of each foot a digital still photograph was obtained. The resultant images were loaded onto a Sony VGN-AR51E laptop computer and using the Ontrack digital software package, the lengths of the four selected metrics were measured.

The measurements for all one hundred feet gave the following results:

Mean dorsal wall length was 73.36mm ± 9.88 SD

Mean hairline to heel length was 100.73mm ± 13.34 SD

Mean bearing border length was 125.23mm ± 16.28 SD

Mean hairline angle was 90.64 degrees ± 2.99 SD

To ascertain whether the Golden Ratio held true for the whole group of feet studied the following calculation was performed:

Mean dorsal wall length = 73.36mm ÷ **3** = 24.45mm, the hairline to heel mean length = 100.73mm ÷ **4** = 25.18mm and the bearing border mean length = 125.23mm

÷ **5** = 25.04mm. Numerically the greater value is 25.18mm and the lower value is 24.45mm, therefore this would imply that Golden Ratio deviated by only 0.77mm for the one hundred feet in this study.

0.55mm error accounted for by dorsal wall length.

0.18mm error accounted for by hairline to heel length.

0.04mm error accounted for by bearing border length.

0.77mm Total

0.55mm ÷ 25mm x 100 = 2.2% deviation in dorsal wall length.

0.18mm ÷ 25mm x 100 = 0.72% deviation in hairline to heel length.

0.04mm ÷ 25mm x 100 = 0.16% deviation in bearing border length.

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Introduction

A practical method of implementing and accommodating what is considered to be the optimum capsular geometrics for an individual hoof conformation has long been the ambition of all concerned with equine locomotor biomechanics. It must be considered as a key factor in the treatment of morphologies, pathologies and diseases.

The laws of physics are well documented and their importance with regard to equine biomechanics should never be underestimated.

Mario Livio, (2002), states that:-

"Physical systems usually settle into states that minimise the energy".

It is this situation that in the natural world, leads to such phenomena as logarithmic spirals, the golden ratio and its founding irrational figure, a figure often referred to by the denotation: *phi,* or possibly by its numerical form of 1.618033. This figure is known to be the most irrational of all irrational numbers, as it has been calculated to 10 million decimal places, so far, and is never repeated and it never ends.

The consideration that mathematics could hold the key to the underlying laws of nature was a proposition first posed by the Pythagoreans between 600 and 400 BC, and the relevance of this to modern farriery may seem remote, possibly even irrelevant. However, recent studies of hoof capsular geometrics and orientation in relation to X, Y and Z axis, have revealed that mathematical ideas and concepts from thousands of years ago may be far more relevant to equine biomechanics than at first given credit.

One if the greatest analogies of the Golden Ratio, was expressed in a statement by Adolph Zeising in 1854 when he stated that:-

"(The Golden Ratio is a universal law) in which is contained the ground-principle of all formative striving for beauty and completeness in the realms of both nature and art, and which permeates as a paramount spiritual ideal, all structures, forms and proportions, whether cosmic or individual, organic or inorganic, acoustic or optical; which finds its fullest realisation."

Albert Einstein, stated that:-

"Mathematics is only a means for expressing the laws that govern phenomena".

The Golden ratio, **3:4:5**, could be considered all of these, mathematics, law and phenomenon.

Could this ratio (Plate 1) possibly hold the key to the optimum proportional lengths that need to be attained in hoof capsular geometrics (in a saggital plane) in order to allow optimum hoof function?

3:- Proportionate length of dorsal wall, hair line to bearing border at toe, mid line dead centre?

4:- Proportionate length of hair line to last point of weight bearing at heel, mid line dead centre?

5:- Proportionate length from last point of weight bearing heel to toe at bearing border, mid line dead centre?

Do proportionate lengths exist in selected measurements of a saggital section of the hoof capsule?

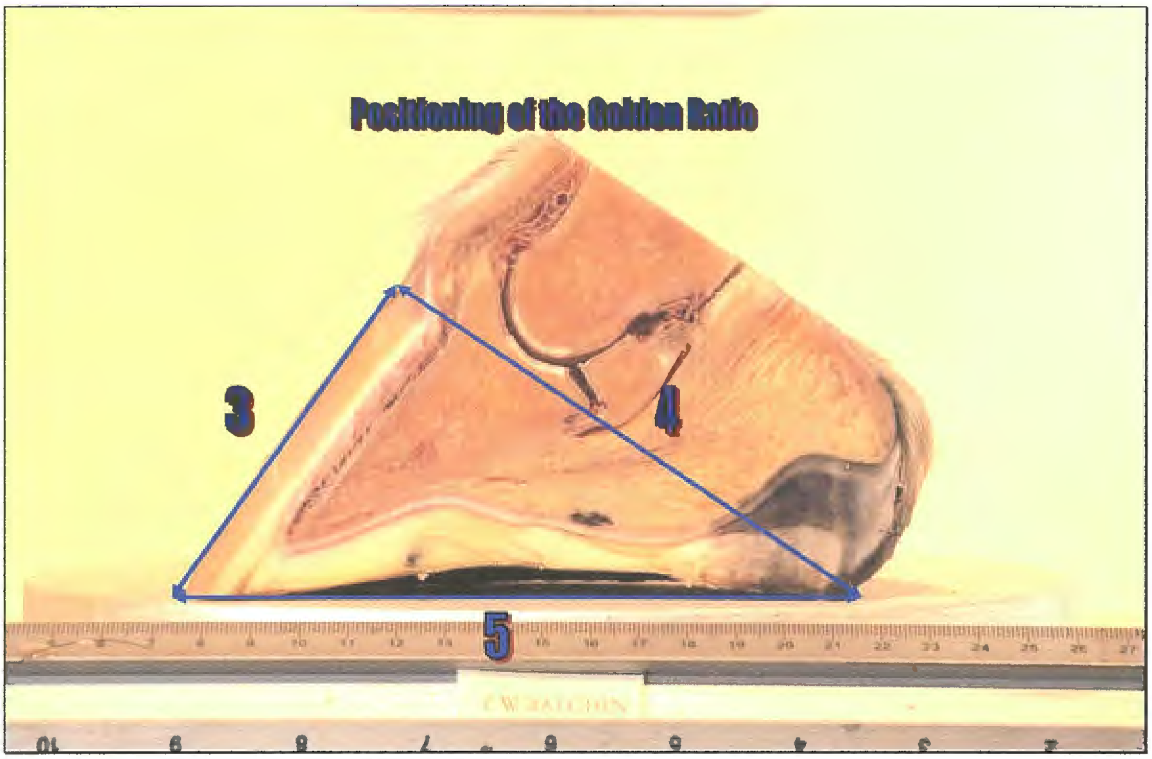

Plate 1:- Positioning of the Golden Ratio.

The reason that these measurements were taken is due in part to observations made in the previous study of 2007-2008, which indicated that when trimmed to M.N.Caldwells (2008) Geometric Proportions trimming protocol, a perfect right angle triangle is formed between the bearing border, the Dorsal wall and a line from the hair line at the Dorsal wall to the last point of bearing at the heel, the 90 degree angle being at the hair line of the Dorsal wall (as seen in Plate 1). This observation was even more impressive when it was realised that in 16 out of the 22 feet, that the right angle triangle had edge lengths in the geometric progression known to form Keplers triangle, which has its roots based in the theorem of Pythagoras and the Golden Ratio.

The Tangent of the right angle triangle was to be found at the hairline of the coronary band and the Sine was found at the bearing border of the toe. This meant that the Cosine was to be found situated at or within millimetres of the last weight bearing point of the heel. The Hypotenuse was formed by the bearing border of the foot and the Legs resulted in the Opposite being formed by the Dorsal wall whilst the

Adjacent was formed by the line from the Tangent to the Cosine or the hairline at the Coronary band to the last weight bearing point of the heel (Plate 1).

When the Golden ratio and Kepler's triangle were positioned onto the foot using Ontrack software, it was interesting to note that the centre of the triangle was located at the point where the Deep Digital Flexor Tendon inserts into the semi lunar crest on the solar surface of the Distal Phalanx, and equally as interesting is the fact that once the triangle and ratio are in position then another Golden ratio and Keplers Triangle are formed. The smaller triangle and ratio are clearly visible on Plate 2 and It would appear that the centre of this triangle represents the Point of Force.

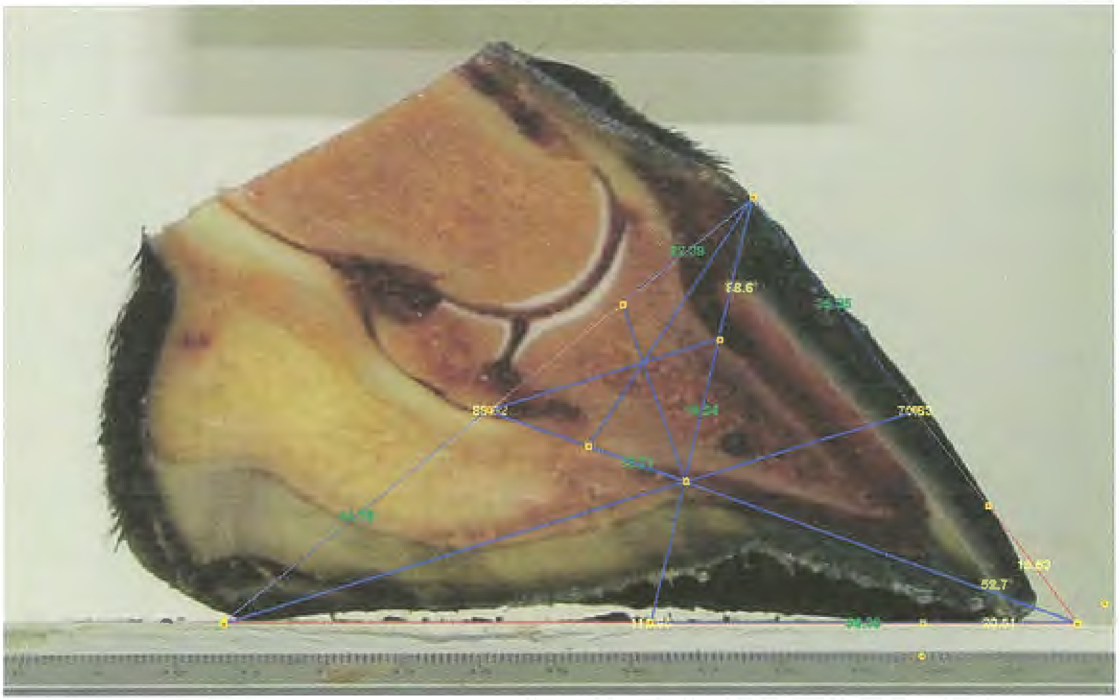

Plate 2:- Saggital section, showing positioning of Keplers Triangle and centres.

As the Dorsal wall angle becomes more acute or more obtuse then the harder it becomes to apply the Golden ratio and Keplers Triangle to the foot because of the disruption in the geometric progression created by the increase in length of the Hypotenuse (bearing border) in relation to the legs (dorsal wall and the line from the hair line at midline dead centre to the last weight bearing point of the heels).

Therefore what should be as close as possible to a 90 degree angle at the hairline at mid-line dead centre increases and this in effect displaces all of the other points of location, and the centres of the triangles no longer represent the point of insertion of the Deep digital flexor tendon or the Point of Force. In this situation careful consideration is required when forming a shoeing protocol and selecting a shoe so as to ensure reinstating the ratio and the triangle.

Study Aims

The study contained within these papers was undertaken in an attempt to ascertain whether or not any significant relationship between optimum trimmed capsular geometrics and the golden ratio truly exist in a randomly selected group of 100 trimmed and saggital sectioned fore feet.

Materials and Methods

It was arranged for 50 pairs of randomly selected forelimbs to be obtained from two sources; a local abattoir run by Tom Goodman & Co and the Warwickshire Hunt kennels. Over a period of four and a half months from June until mid October 2010 the limbs were collected. They arrived in varying states of morphological form, some with shoes on and recently shod, whilst others were in a total state of neglect (Plate 3). The actual causes of death of most of the animals were unknown however in some cases it was quite clear, they included laminitis (Plate 5), broken limbs, (Plate 6) and varying situations that only became apparent on dissection such as pathology of the distal sesamoidean bone,(Plate 7). Some individual cases were not obvious to the naked eye, and only revealed their true extent upon digital enhancement, for example Plate 8 shows a case of a fractured 2nd Phalanx.

Plate 3:- Random Foot Selection.

The feet varied in size, breed and age, from a miniature Shetland to a 17hh show hunter, from a 3 month old thoroughbred foal whose demise was bought about as a result of a collision with a power cable to a 21 year old brood mare who had to be

Plate 4:- Frozen limbs prior to removal of hoof capsule.

Plate 5:- Lamanitic hoof capsule.

destroyed on ethical grounds due to an infection of the carpus which had resulted in a permanent lameness, the synovium had the consistency and colour of mud and smelt rancid. The articular surfaces of the carpal bones were in an advanced state of degeneration, and so the parties involved deduced that the situation had been allowed to progress for far to long.

Plate 6:- Fracture line at mid third metacarpal bone.

The above picture (Plate 6) was the result of a 10 year old hunter placing its Forefoot down a Badgers earth whilst out on exercise and the actual break is situated at mid third metacarpal. Limb removal was achieved by severing the flexor tendons only as the extensor tendons had already been severed.

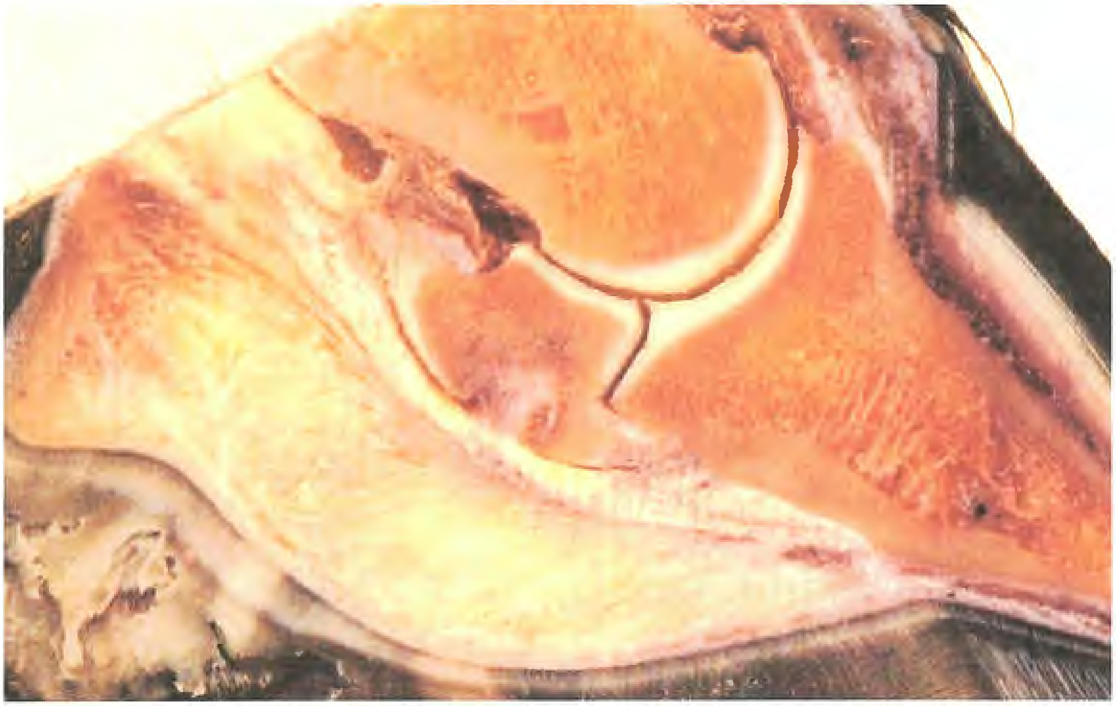

Plate 7:- Saggital section showing large lesion in distal sesamoidean bone.

Plate 8:- Saggital section showing fractured second phalanx.

The feet attached to these limbs were trimmed to a strict protocol based on M.N.Caldwells Geometric proportions, which encompasses M.Savoldi's Uniform sole thickness, D.Ducketts Dot, bridge and pillars and the NVQ level 3 trimming protocol.

Trimming Protocol

The trimming protocol was carried out in the order as follows and the actual process took on average 50 minutes per foot which equates to 83.5 hours just to trim the feet:-

Uniform Sole Thickness (UST)

This involved the removal of all exfoliating sole, including the sole callus which is situated at the toe area between 10 & 2 o'clock or Ducketts Pillars, and from seat of corn to seat of corn both medially and laterally. The White line is trimmed to expose its junction on the outer edge with the wall and on its inner edge with the sole.

The bars are exfoliated and sculpted to reveal the live bars and the frog is trimmed to live frog in an appropriate manner that would engage load bearing forces at static mid stance once excess wall has been removed perpendicular to the long axis.

Geometric Proportions

Geometric capsular proportions are imposed on the foot by removing excess wall at the bearing border in a horizontal plane, perpendicular to the long axis, ensuring that the heel buttresses at the bearing border are trimmed down to the widest point of the frog so that the toe to heel ratio (Plate 9) measures 3-1. The bearing border is then reduced in length so that the vertical height of the dorsal wall at mid line dead centre is equal to the distance from the tip of the toe to the widest point of the foot when viewed from the solar surface. The dorsal wall is dressed from quarter to quarter so that when viewed from the solar surface its width forms a uniform distance from the white line interface. The medial and lateral surfaces of the frog are trimmed at a 90 degree angle to their corresponding bars and the medial and lateral sulci are trimmed to the full depth in conjunction with the bars, the central sulci is trimmed at a 90 degree angle to the medial and lateral sulci in an appropriate manner. The length of the frog is trimmed to expose and identify the true point of the frog.

Exfoliating solar horn should be removed from the seats of corn both medially and laterally including the sole callus, revealing the true point of frog down to live horn, this process is essential to reveal the true solar plane.

Plate 9:- Toe to heel ratio of Three to One.

Foot Mapping

Foot mapping is in effect a tool that enables a practioner to assess capsular proportions in relation to an individual conformation type. It is carried out using a fine tip dryline marker and a straight edge, such as a brass rule (Plate 10).

On the solar surface of the foot a straight line is drawn through the central sulci down the length of the frog, through the true point of frog and through the centre of the toe, bisecting the foot. Parallel to this both medially and laterally two lines are drawn from the last weight bearing points of the heels dorsally through the toe pillars.

The toe pillars and the last weight bearing points of the heels are connected by the drawing of a line box built on these four points of reference and then they are connected by two diagonal lines drawn from toe pillars to diagonal last weight bearing points of heels. A line is drawn medial/laterally across the widest point of the foot, this line should converge with the two diagonal lines and the line that bisects the foot medial/laterally, the point where these four lines converge is known as the 'centre of rotation' (COR) and it is adjacent to Ducketts Bridge, which is a theoretical external reference point on the solar aspect of the foot that relates directly to the positioning of the centre of rotation. The distance from this point to the centre of the toe should be equal to the distance from the centre of the toe at the bearing border dorsally to the hair line at mid line dead centre. This specific measurement should be checked using a pair of J H Forge calibrated dividers.

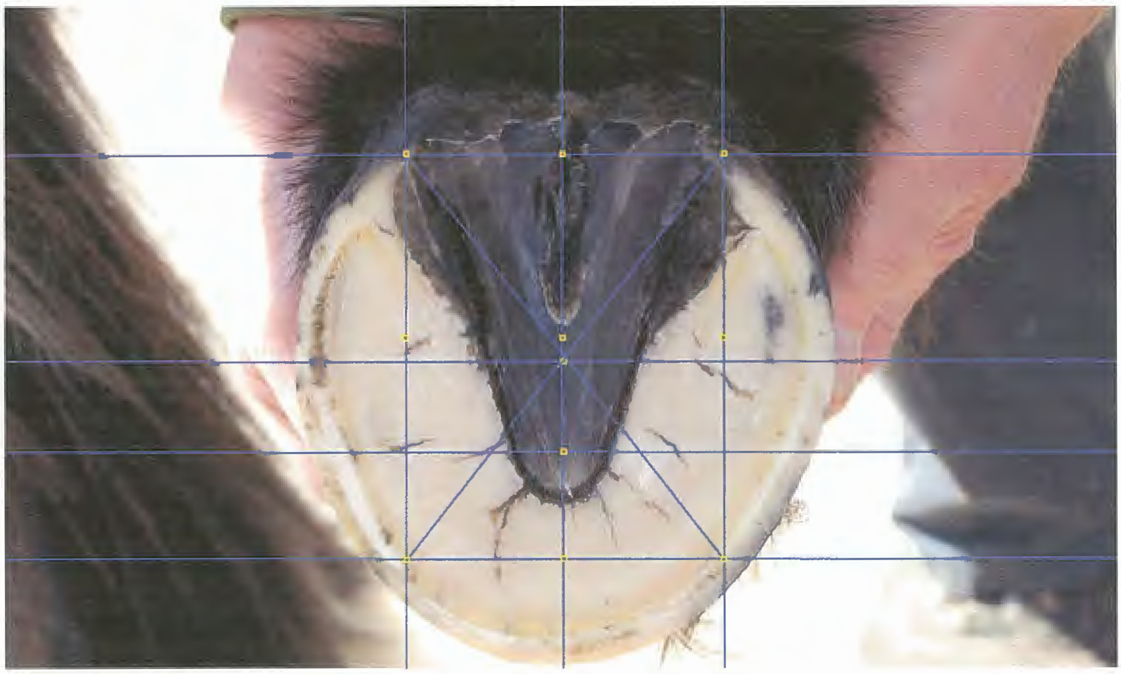

Plate 10:- Digital foot mapping.

Live Horn

As horn is exfoliated/trimmed it undergoes a number of characteristic changes and this process is virtually the same for the frog, wall, bars and sole. Firstly, a layer of foreign debris is removed. Then the first true horn to be removed is a layer of loose exfoliating horn. This is followed by a layer of compact but still exfoliating horn. The next layer to be removed tends to have almost a powdery texture to it. Finally live horn is exposed, which is not removed or penetrated, and is characterised by its waxy appearance and texture.

Where possible compromised and damaged structures should be trimmed back to live horn.

Trimming the Bearing Border

Prior to the removal of any horn from the bearing border a thorough visual inspection should be undertaken to observe any morphologies or pathologies such as flaring, tearing or compression of the structures. The solar plane should be assessed in relation to the long axis of the limb. Excess horn should be carefully removed from toe to heel parallel to the live sole both medially and laterally being careful not to invade the live sole. The medial and lateral heels are trimmed to the widest point of the frog which should correspond with the most caudal aspect of the central sulci.

The rasp should be used to remove the last of the excess horn and this should be carried out with a firm even pressure, at this stage being particularly careful not to create any deviation of the horizontal solar plane in relation to the long axis of the limb.

The width of the dorsal wall is determined by the width of the inner border of the white line interface at the medial and lateral quarters, so from these reference points the dorsal wall thickness should be dressed to create a uniform wall thickness.

The next stage is for the limb to be elevated cranially and the foot to be dressed forward to what has now become the peripheral border of the solar surface, this should be carried out with a mind set based on conservancy and preservation.

Limb removal and Foot Dissection

Once the feet had been trimmed they were individually placed in labelled freezer bags and then frozen in a Scandinova chest freezer (Plate 11). Whilst in a frozen state the feet were removed from the limb proximal to the coronary band with the use of a Dewalt, DW738, electric band saw (Plates 13 and 14). The feet were then halved with a single saggital cut at midline dead centre, this cut was made along the same bisecting line that forms part of the foot mapping process depicted in Plate 10.

Plate 11:- Feet in frozen state in chest freezer.

The cut was made dorsal/palmer so as to bisect the centre of the toe the true point of the frog and the length of the frog through the central sulci, exiting the foot between the bulbs of the heels. As part of the foot mapping process a measurement was taken from the bisecting line, at 90 degrees to the widest part of the solar surface of the foot, laterally. This measurement corresponded to the positioning of the calibrated cutting gauge on the working surface (Plate 12) of the bandsaw to ensure that the cut is in the correct position and accurately made with no divergence.

Plate 12:- Cutting gauge on working surface of bandsaw.

All work performed with the band saw was done with the feet in a frozen state. This was to eradicate the problems associated with distortion, an inconvenience which would compromise the results of the study.

Whilst still frozen, the lateral side of each foot was cleaned using boiled water and a soft brush to remove any tissue fibres, and then dried using a clean soft cotton towel . This process has to be carried out rapidly because if it is not then the boiled water will quickly freeze to the surface of the internal structures. This traps any tissue fibres and in the process creates a reflective surface. This is not good for photographic purposes. The next stage was for the lateral half of each foot to be placed in the calibrated photographic box (Plate 15) and photographed using a Nikon D80 DSLR camera, mounted on a Giottos MTL9351B Tripod.

Once this had been carried out and the lateral halves measured, each foot was wrapped in a labelled freezer bag and placed back in the freezer in case it was needed for future reference. This process was carried out on all 100 feet.

Plate 13:- Dewalt DW738 bandsaw.

Plate 14:- Small selection of feet used in study.

The Calibrated Photographic Box

This was made especially for this study. It consists of a base, a back, and two sides. It measures 30cm square with an elevated plinth of 15cm diameter to enable better visual access to the subject. The back of the box has two brass straight hooks screwed into it approximately 24cm up from the base. These hooks are to enable the subject to be assigned a label, using a rectangular piece of white board measuring 15cm x 9 cm with two 8mm holes drilled in it on the top long edge 2cm from each end. The rectangular piece of white board had the subject number written on it using a dry line marker and then it was positioned in the back of the calibrated box, suspended from the two brass hooks. In this way, the rectangular piece of white board together with the individual subject identifying number was clearly visible in the background of each photograph.

The box and the elevated plinth were painted matt white using Leyland white undercoat. This clearly enabled the subject to be easily distinguished from the surrounding environment without glare, this is essential for the accurate positioning of the points of measurement.

The reason for the box being painted matt is so that there was minimal glare from the surrounding light sources. This also reduces flashback from the camera flash as this was set to automatic.

Plate 15:- The Calibrated photographic box, build specifically for this study.

Computer analysis using Ontrack Software:

Once all of the required pictures were on the memory card of the camera this was removed from the camera and loaded onto a Sony Vaio VGN-AR51E laptop so that the pictures could be cropped to size and enhanced using Windows Vista and labelled in preparation for loading onto the Ontrack software.

Using Ontrack, 100 individual files were created (one for each foot) and a picture of each foot was loaded into its corresponding file, so that each file contained one picture. Each picture had measurements applied at the same specific points, but prior to this they had to be calibrated using the aluminium rule which is positioned in the base of the photographic box and this is clearly visible in all pictures.

Points of measurement:

Four specific measurements were plotted using the Ontrack software, these are as follows:-

1:- Angle at the hair line, Tangent, (mid line dead centre).

2:- Dorsal wall hair line to bearing border at toe (DW).

3:- Dorsal wall hair line, to last point of weight bearing at heel (DWH to HEEL).

4:- Last point of weight bearing at heel to toe at bearing border (T to H).

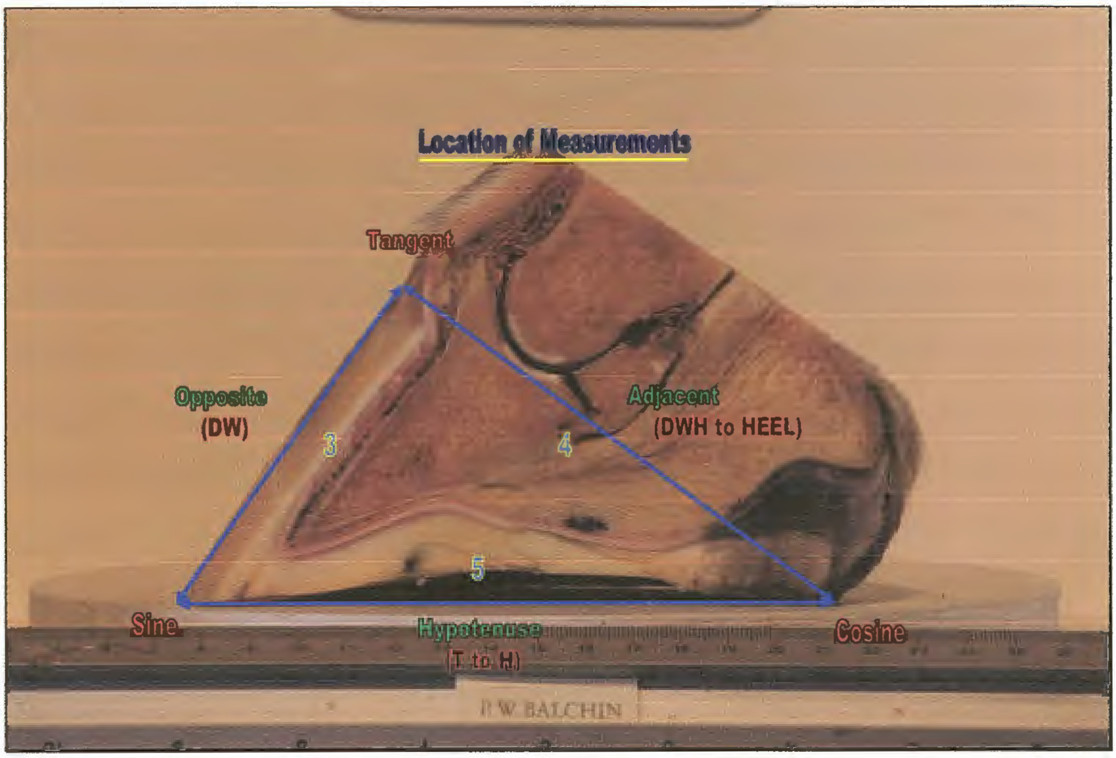

Plate 16:- Saggital section, showing points of measurements.

Once all of the measurements had been collected then the individual edge lengths of the triangle had to be divided by the corresponding figures to ascertain the accuracy of the proportionate lengths. The Dorsal wall (Opposite) was divided by three, the Dorsal wall hairline to heel (Adjacent) was divided by four and the Bearing border (Hypotenuse, T-H) was divided by five. The closer these resultant figures from each individual foot, the greater the accuracy of the relationship with the Golden ratio.

Results

Angle at hairline, an angle of 90 degrees was found in 59% of the feet investigated in this study with a further 7% being within 1 degree of 90 degrees, that being 89-91 degrees (Figure 1).

The most acute hair line angle was found to be 82.4 degrees and that was in foot 44A whilst the most obtuse hairline angle was found to be 100.4 degrees which was in foot 27A, and thus these gave the range of the data.

Foot 27A, although not forming a right angle triangle, its proportionate edge lengths varied by only 2.5 millimetres, whilst the edge lengths of foot 44A varied by only 2.2 millimetres.

The greatest variation in the proportionate edge lengths was found in foot 32A and this foot showed a variability of 4.89 millimetres, however the hairline angle was a perfect 90 degree angle.

Encompassing all 100 feet the Mean hairline angle was 90.644 degrees whilst the Median was 90.150 degrees and the Mode was revealed to be 90 degrees with a Standard deviation of 2.993 degrees and a Variance of 8.959 degrees.

As would be expected paired feet data sets bore a strong relationship to each other unless physiological or morphological influences were considered a factor such as with pair of feet 3AB.

The largest foot in the study was foot 21B which had a hairline angle of 90.2 degrees and the proportionate edge lengths varied by only 3.47 millimetres.

The smallest foot in the study was foot 49A, which belonged to a miniature Shetland pony, this foot had a hairline angle of 84.8 degrees which although considered as an acute hairline angle this foot revealed a variation in progressive edge lengths of only 1.29 millimetres.

Relative proportionate edge lengths with a limited amount of variability show a far greater degree of consistency regardless of the hairline angle.

This would bring about a new strength to the term, "Disproportionate Capsular Geometrics".

Variability in progressive edge lengths bore a direct correlation to foot size (Table1).

Summary of Results

Dorsal Wall Mean Length	(Opposite)	=	73.36mm /3 = 24.45mm
Hairline to Heel Mean Length	(Adjacent)	=	100.73mm /4 = 25.18mm
Bearing border Mean Length	(Hypotenuse)	=	125.23mm /5 = 25.04mm

If the Data set is thus divided by the 3-4-5 (because this represents the Golden Ratio) the closer in numerical value the final figures, the greater the accuracy of the ratio.

/3 = 24.45mm

/4 = 25.18mm

/5 = 25.04mm

Thus: the numerically greater value is 25.18mm and the lower is 24.45mm, therefore; 25.18mm – 24.45mm = 0.73mm.

This figure (0.73mm) indicates that in over 100 feet (N=100), the Golden Ratio deviated by less than three quarters of a millimetre (0.73mm), and for each length measurement the percentage deviation from an absolute mathematical ratio was:

0.55mm ÷ 25mm = 2.2%

0.18mm ÷ 25mm = 0.72%

0.04mm ÷ 25mm = 0.16%

All measurements in Millimetres unless otherwise stated.

(The closer the final three figures are to each other in the three right hand columns of each row, the closer the conformity to the golden ratio).

Key to Table 1:-

A = Right foot.

B = Left foot.

* = Laminitic pathology present

† = Distal sesamoidean pathology present.

Foot	Angle at hairline Degrees	Dorsal wall length	Hairline To heel length	Bearing Border length	Dorsal Wall Divide 3	Hairline To heel Divide 4	Bearing Border Divide 5
1A	90.3	80.9	105.23	132.32	26.70	26.31	26.46
1B	90.4	79.76	105.23	132.6	26.59	26.31	26.52
2A	90	82.25	118.64	144.26	27.42	29.66	28.85
2B	90	85.36	119.76	147.04	28.45	29.94	29.41
3A*	94.9	86.0	106.71	142.61	28.67	26.68	28.52
3B	90.1	81.57	106.61	134.41	27.19	26.65	26.88
4A	90.06	78.61	112.82	138.15	26.20	28.21	27.63
4B	90.06	78.07	108.39	134.24	26.02	27.10	26.85
5A	86	89.43	126.0	149.4	29.81	31.50	29.88
5B	85	92.45	129.4	151.98	30.82	32.35	30.40
6A	91.7	85.48	118.18	147.9	28.49	29.55	29.58
6B†	90.5	85.71	116.86	145.53	28.57	29.22	29.11
7A	89.5	76.38	99.85	125.19	25.46	24.96	25.04
7B	90.6	73.93	99.81	125.15	24.62	24.95	25.03
8A	89.3	75.51	105.22	128.80	25.17	26.31	25.76
8B	89.5	80.31	107.02	133.24	26.77	26.76	26.65
9A	88	51.96	78.45	92.61	17.32	19.61	18.52
9B	87.4	53.23	76.35	90.34	17.74	19.09	18.15
10A†	90.9	78.92	106.62	133.65	26.31	26.66	26.73
10B†	90	82.39	105.88	135.14	27.46	26.47	27.03
11A*	92.5	75.6	97.23	125.78	25.20	24.31	25.16
11B*	92.1	80.64	96.83	128.3	26.88	24.21	25.66
12A	90.7	58.79	86.17	104.89	19.60	21.54	20.98
12B	83.6	61.11	84.71	98.76	20.37	21.18	19.75
13A*†	93	72.61	101.98	127.98	24.20	25.50	25.60
13B*	90	67.67	103.85	124.0	22.56	25.96	24.8
14A	90	70.48	109.39	130.14	23.49	27.35	26.03
14B	88	74.74	108.46	129.50	24.91	27.12	25.9
15A*	89.5	67.1	94.83	115.37	22.37	23.71	23.07
15B*	92.8	65.2	100.89	122.75	21.73	25.22	24.55

16A	90	86.38	118.94	146.99	28.79	29.74	29.4
16B†	90.1	87.81	119.2	148.18	29.27	29.8	29.64
17A	91.1	77.79	111.21	137.23	25.93	27.8	27.45
17B	90.9	78.03	112.06	137.21	26.01	28.02	27.44
18A	90.2	82.51	109.77	137.52	27.5	27.44	27.5
18B	90	85.23	108.68	138.13	28.41	27.17	27.63
19A	90	72.1	104.18	126.42	24.03	26.05	25.28
19B	90	71.62	103.4	125.76	23.87	25.85	25.15
20A	90.3	66.92	97.73	118.77	22.31	24.43	23.75
20B	90.2	70.25	93.57	117.2	23.42	23.39	23.44
21A	90	93.82	115.67	149.23	31.27	28.92	29.85
21B	90.2	96.77	115.14	150.65	32.26	28.79	30.13
22A	90.1	67.49	102.82	123.06	22.5	25.71	24.61
22B	91.6	69.12	98.46	121.88	23.04	24.62	24.38
23A	91.5	65.43	103.88	124.17	21.81	25.97	24.83
23B	90.2	67.06	104.01	123.97	22.35	26.00	24.79
24A	94.2	63.17	88.78	112.66	21.06	22.20	22.53
24B	98.9	62.97	91.04	118.45	20.99	22.76	23.69
25A*	90.8	69.8	104.81	126.78	23.27	26.2	25.36
25B*	93.2	72.3	98.99	125.81	24.10	24.75	25.16
26A	98.1	63.25	91.22	118.14	21.08	22.81	23.63
26B	96.7	65.92	95.41	122.15	21.97	23.85	24.43
27A*	100.4	72.06	99.01	132.6	24.02	24.75	26.52
27B*	95.5	74.83	98.01	128.92	24.94	24.50	25.78
28A	90	72.11	105.39	127.39	24.04	26.35	25.48
28B	90.9	71.41	96.52	120.94	23.8	24.13	24.19
29A	90.1	74.25	94.56	120.01	24.75	23.64	24.00
29B	90.1	69.91	90.1	114.14	23.30	22.53	22.83
30A	93.9	80.91	107.2	138.76	26.97	26.80	27.75
30B	94.9	80.17	100.88	134.10	26.72	25.22	26.82
31A	98.5	67.53	90.79	120.91	22.51	22.70	24.18
31B	95.4	64.77	85.80	112.30	21.59	21.45	22.46
32A	90	73.02	116.92	137.84	24.34	29.23	27.57
32B	90.4	71.03	115.46	136	23.68	28.87	27.20
33A	90	76.93	90.48	118.77	25.64	22.62	23.75
33B	90	78.68	97.71	125.49	26.23	24.43	25.10
34A*	97	67.71	94.81	123.06	22.57	23.70	24.61
34B*	97	72.09	94.73	125.81	24.03	23.68	25.16
35A	90.5	63.57	92.07	112.3	21.19	23.02	22.46
35B†	90.4	65.28	90.88	112.3	21.76	22.72	22.46
36A*	90.4	70.01	96.25	119.37	23.34	24.06	23.87
36B*	86.8	73.41	94.75	116.62	24.47	23.69	23.32
37A	90.5	71.62	97.45	121.42	23.87	24.36	24.28
37B	90.5	73.9	98.14	123.37	24.63	24.54	24.67
38A	90.9	75.07	102.08	127.7	25.02	25.52	25.54
38B	90	74.7	99.31	124.29	24.9	24.83	24.86
39A	90.6	73.66	102.41	126.78	24.55	25.60	25.36
39B	90.2	77.02	99.57	126.12	25.67	24.89	25.22

40A	90	60.01	74.47	95.38	20.00	18.62	19.08
40B	90	57.91	69.56	90.47	19.30	17.39	18.09
41A	90.1	77.41	119.06	142.46	25.8	29.77	28.49
41B	90.1	75.1	112.91	135.7	25.03	28.23	27.14
42A*	88	63.05	88.3	106.71	21.02	22.08	21.34
42B*	88.7	61.86	85.66	104.51	20.62	21.42	20.90
43A	90	76.84	100.77	126.12	25.61	25.19	25.22
43B	90.1	73.47	103.08	126.41	24.49	25.77	25.28
44A	82.4	75.35	106.9	122.43	25.12	26.73	24.49
44B	86.7	76.36	103.53	125.07	25.45	25.88	25.01
45A	90.8	83.47	110.41	139.36	27.82	27.60	27.87
45B	90.5	81.91	113.57	140.31	27.30	28.39	28.06
46A	92.3	89.97	124.97	156.92	29.99	31.24	31.88
46B	90.6	88.6	116.83	137.04	29.53	29.21	27.41
47A	90	81.53	111.92	139.04	27.18	27.98	27.81
47B†	90	83.88	109.36	137.85	27.96	27.34	27.57
48A*	90.4	65.97	83.81	107.06	21.99	20.95	21.41
48B*	90.1	63.68	83.25	104.91	21.23	20.81	20.98
49A	84.8	43.68	53.93	66.35	14.56	13.48	13.27
49B	83.7	43.93	56.12	67.37	14.64	14.03	13.47
50A	87.9	57.79	82.6	99.04	19.26	20.65	19.81
50B	87.6	61.63	84.06	102.46	20.54	21.02	20.49

Table 1:- Measurements of Edge Lengths.

Basic Statisitics

Descriptive Statistics: ANGLE AT HAIR LINE

Variable	Total Count	N	N*	Mean	SE Mean	StDev	Variance	CoefVar
ANGLE AT HAIR LINE	100	100	0	90.644	0.299	2.993	8.959	3.30

Variable	Minimum	Q1	Median	Q3	Maximum	Range	Mode
ANGLE AT HAIR LINE	82.400	90.000	90.150	90.900	100.400	18.000	90

Variable	N for Mode	Skewness	Kurtosis
ANGLE AT HAIR LINE	20	0.63	2.29

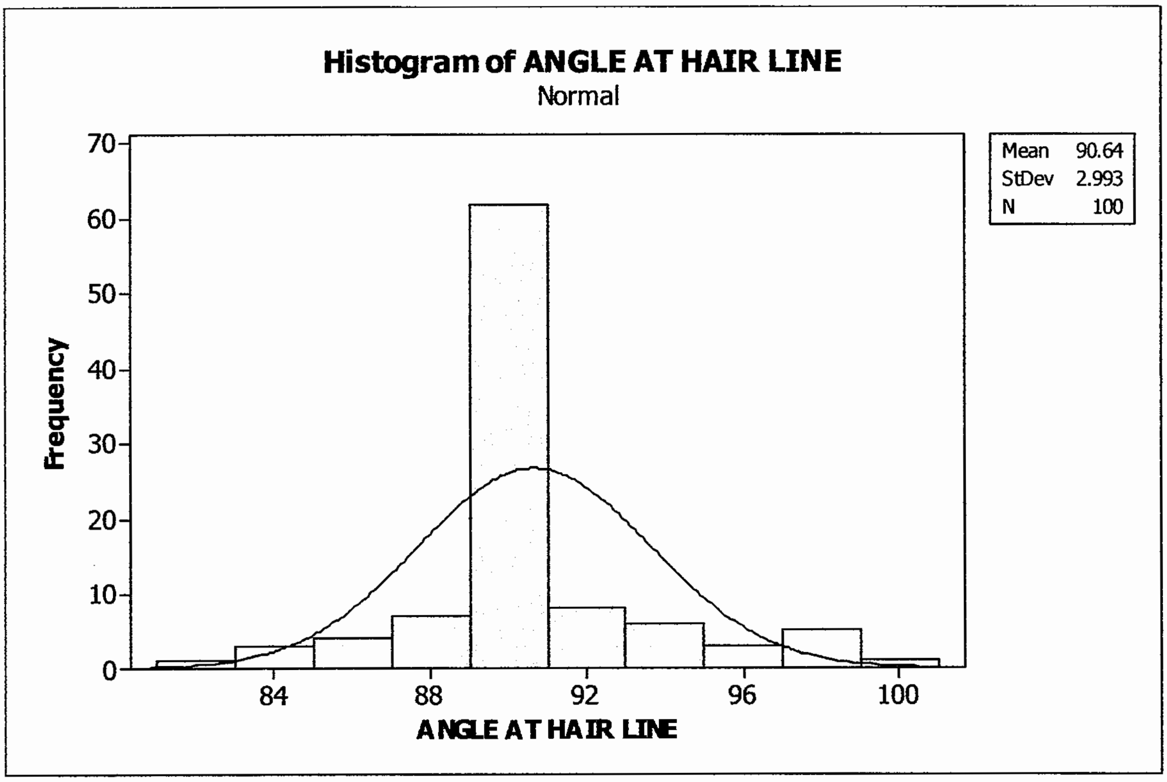

Figure 1:- Histogram of Angle at Hair Line.

Descriptive Statistics: DORSAL WALL

```
               Total
Variable       Count    N  N*    Mean  SE Mean  StDev  Variance  CoefVar
Minimum
DORSAL WALL      100  100   0  73.361    0.988  9.880    97.614    13.47
43.680

                                                                N for
Variable          Q1  Median      Q3  Maximum   Range   Mode   Mode  Skewness
DORSAL WALL   67.070  73.780  80.150   96.770  53.090  71.62      2     -0.37


Variable      Kurtosis
DORSAL WALL       0.64
```

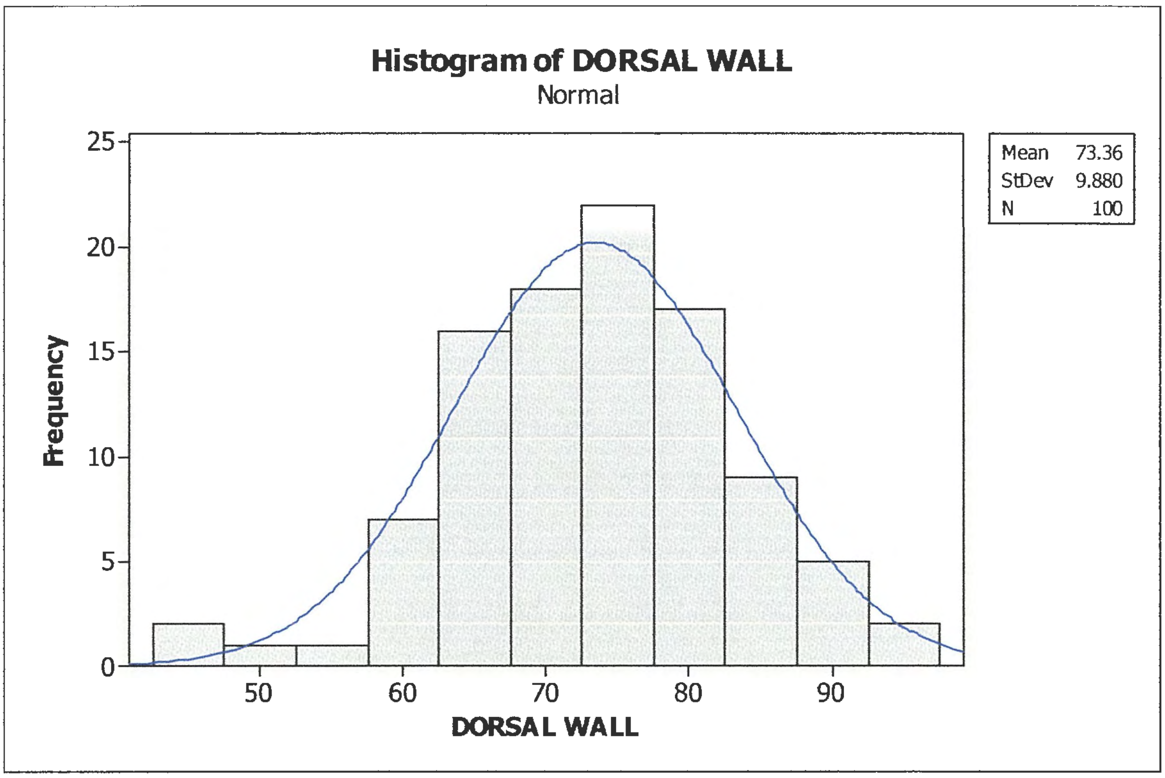

Figure 2:- Histogram of Dorsal wall length.

Descriptive Statistics: HAIR LINE TO HEEL

Variable	Total Count	N	N*	Mean	SE Mean	StDev	Variance	CoefVar
HAIR LINE TO HEEL	100	100	0	100.73	1.33	13.34	178.08	13.25

Variable	Minimum	Q1	Median	Q3	Maximum	Range	Mode
HAIR LINE TO HEEL	53.93	94.60	102.03	109.19	129.40	75.47	105.23

Variable	N for Mode	Skewness	Kurtosis
HAIR LINE TO HEEL	2	-0.80	1.67

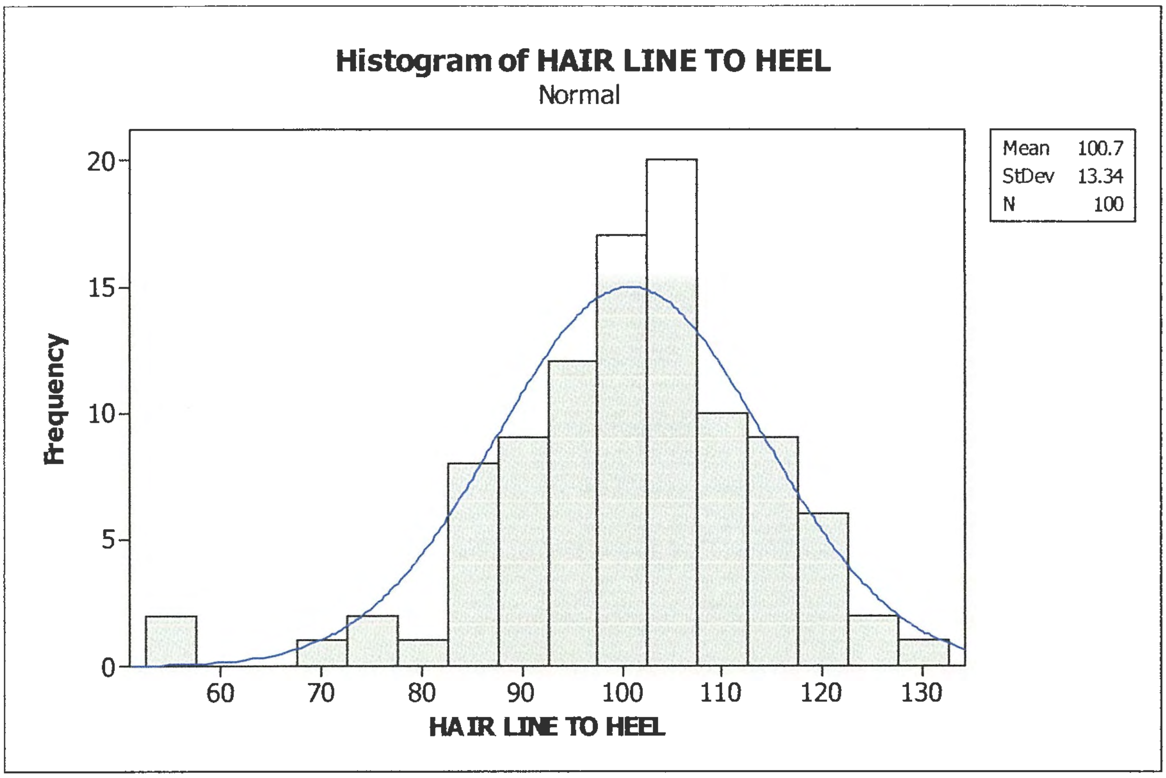

Figure 3:- Histogram of Hairline to Heel length.

Descriptive Statistics: Bearing Border

```
              Total
Variable      Count    N  N*     Mean  SE Mean  StDev  Variance  CoefVar
Minimum
TOE TO HEEL     100  100   0  125.23     1.63  16.28    265.19    13.00
66.35

                                                                N for
Variable          Q1  Median      Q3  Maximum  Range   Mode   Mode  Skewness
TOE TO HEEL   118.77  125.97  136.78   156.92  90.57  112.3      3     -1.06


Variable      Kurtosis
TOE TO HEEL       2.20
```

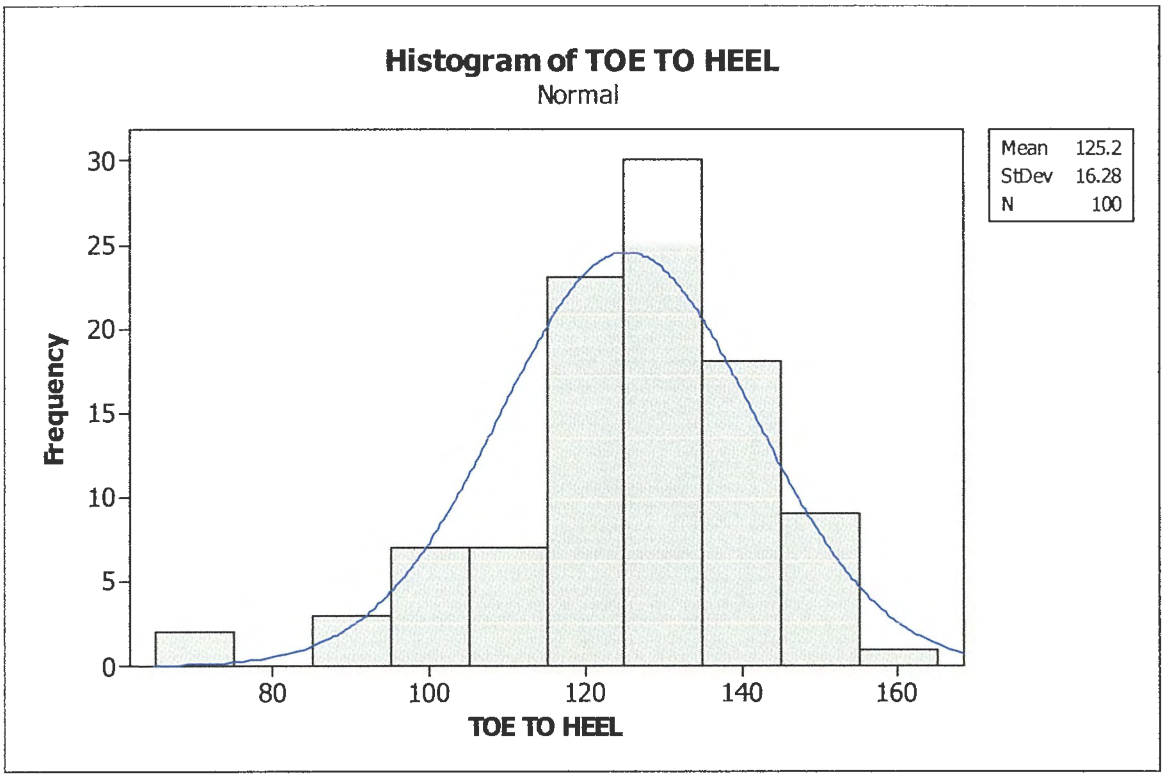

Figure 4:- Histogram of Bearing Border length.

Discussion of Results

The hairline angle of 66% of the feet studied in this investigation fell within a range of one degree of the 90 degree hairline angle, and 59% formed the perfect 90 degree angle. This implies that of the 100 feet contained within this study 59 of them formed a perfect right angle triangle (Table 1 & Figure 1). The feet that did not form the perfect 90 degree angle could be said to have been influenced by varying forms of pathological changes but not necessarily morphological changes, due in part to the fact that morphological changes are accommodated for in the trimming process.

It may be that one factor influencing the geometrics of the individual hoof capsules contained within this study that did not form a right angle triangle were pathological changes created as a result of varying degrees of, for example laminitis.

The mean hairline angle was revealed to be 90.644 degrees with a standard deviation of 2.993 degrees and a range of 18 degrees which when considered over 100 feet gave a non-normal distribution (Figure 1 & Appendix IV) but with an obvious mean, median and mode very close to each other. In other words, a great many of the hairline angles were at 90 degrees and more so than would have occurred in a normal distribution.

It appears that regardless of hoof size and hairline angle, proportionate edge lengths occur (foot 44B). In this foot despite not having a 90 degree hairline angle, proportionate lengths occurred. This should be theoretically impossible and therefore is possibly due to compounding slight measurement errors.

The relationship between proportionate edge lengths and hairline angle is inconsistent (Table 1), this is because for the whole data set there were some capsular abnormalities. 59% formed the perfect right angle triangle then another 7% were ± 1 degree.

Of the 100 feet contained within this study a number revealed upon digital enhancement varying forms of pathology, these included lamanitic and distal sesamoidean changes.

If these were taken out and studied as a sub-section then it would be interesting to see by how much they have influenced the results.

Dorsal wall length varied considerably and this would be as a result of the considerable difference in foot size with a range of 53.090mm, however the mean dorsal wall length was 73.361mm whilst the median was 73.780mm (Figure 2).

John Eddison (1999) states that:-

"The coefficient of variation is simply the ratio of standard deviation to mean and it is often expressed as a percentage".

The coefficient of variation for the dorsal wall, which as with the rest of the analysis for the results was analysed using Minitab 15, was revealed as 13.47%, this is an acceptable figure for a biological measure.

The descriptive statistics for the hair line to heel measurements (Figure 3) revealed a minimum of 53.93mm which was one of a pair of feet belonging to a Miniature Shetland pony (feet 49 A & B). The maximum was found to be 129.40mm (foot 5B) which was one of a pair of feet belonging to a 7 year old advanced Show Hunter (Appendix III). The median hairline to heel measurement was calculated to be 102.03mm and the coefficient of variation was 13.25%.

The bearing border descriptive statistics (Figure 4), reveal a range of 90.57mm with the minimum being 66.35mm (foot 49A) and the maximum being 156.92mm (foot 46A). The mean bearing border measurement was 125.23mm with a standard deviation of 16.28mm and a coefficient of variation of 13%, which is surprisingly close to the 13.47% for the dorsal wall and the 13.25% for the hairline to heel.

The probability plot of angle at hair line (Appendix IV) and the summary for angle at hairline (Appendix VI) reveal an non-normal distribution with a P-Value of <0.005. An Anderson-Darling Normality Test was used to used to achieve this result and if it is considered that 66% of the feet in this study presented a hairline angle that was within one degree of the 90 degree angle and the mean hairline angle was 90.64 degrees this P-Value does not correlate with the actual measurements. It could be a consideration that the Anderson-Darling Normality test was not the most appropriate test to use in this situation.

The probability plot of the dorsal wall (Appendix IV) and the summary for dorsal wall (Appendix VI) reveal a normal distribution and a P-Value of 0.761, and hairline to heel probability plot (Appendix V) and the summary for hairline to heel (Appendix VII) reveal a normal distribution with a P-Value of 0.072. The bearing border probability plot (Appendix V) and the summary for bearing border (Appendix VII), reveal a non-normal distribution and a P-Value of <0.005. The reason for this being that not only is this the greatest measurement but it is also the most varied, with a range of 90.57mm a maximum of 156.92mm and a minimum of 66.35mm (Figure 4 & Appendix VII).

The probability plot of dorsal wall divided by 3 (Appendix VIII) reveal two feet that are not within the common range, these are feet 49A & B belonging to the same miniature Shetland pony previously mentioned. It is an interesting observation that as with most situations relating to miniature Shetland ponies, even when no longer living they can still not be relied on to conform.

The probability plot of bearing border divided by 5 (Appendix X) clearly show that not only are feet 49A & B out of the common range but also at the top of the plot another two feet are out of the common range and these are not a pair but two individual feet. The reason these two feet are out of the common range is that when the individual bearing border measurement is divided by 5 these are the only two feet to have a value greater than 30mm with foot 21B measuring 30.13mm and foot 5B measuring 30.40mm.

In a number of the feet, changes to the distal sesamoidean bone were revealed upon digital enhancement (Plate 7). These changes varied in degree of severity. They did however appear to be more prevalent in the feet that could be described as suffering from disproportionate capsular geometrics as a result of pathological influence.

When considering the feet whose hair line did not form a 90 degree angle it was remarkable how close the edge lengths were to forming progressive proportionate edge lengths regardless of the variability of the hairline angle.

The subject of the Golden ratio has both frustrated and inspired mankind equally, religions have been built around it and lifetimes devoted to its study but it should be borne in mind that when the ancient Greeks invented the study of Triangles known as Geometry the pathway to the discovery of this ratio was laid.

Mario Livio (2002) states that:-

"The Golden ratio is a product of humanly invented geometry. If geometry had not been invented then we might have never known about the Golden Ratio".

Whilst Darcy Wentworth Thompson (1860-1948) claims that:-

"In some cases at least, the forms of living things, and of the parts of living things, can be explained by physical considerations, and to realise that in general no organic form exists save such as are in conformity with physical and mathematical laws".

On a subject of this magnitude it would be easy to go down the route of Paradomeia and visualise this ratio in all manner of situations.

Plato (428-348BC) referred to the Golden Ratio as a "Continuous Geometric Progression".

Difficult as it may be to comprehend the Golden ratio is an irrational number. In the fifth century B.C the Greek mathematician Hippasus of Metapontum discovered that the Golden ratio is neither a whole number (1,2,3…) or a ratio of two whole numbers such as a fraction (1/2, 2/3, ¾…). Whole numbers and fraction are termed rational numbers.

Hippasus discovery is termed "Incommensurability" which means that the value is incapable of being measured, judged or considered comparatively. The Collins English Dictionary describes this as "Unrelated to another measurement by Integral multiples" or "Not having units of the same dimension". This situation means that if a line were to be sectioned into a Golden ratio, the three sections including the complete line, a common value is unachievable. The same can be said for the diagonal of a square in relation to its side or the relationship of the side of a Pentagon in connection to its diagonal. This means that it is not possible to attain a common measurement.

As a result of the discovery of Incommensurability one of the facts that is certain is that the Golden ratio cannot nor shall it ever be isolated to a specific quantative value or measurement. At this stage one could be forgiven for questioning whether or not the Golden ratio truly exists at all.

In his greatest work known as "On Growth and Form" Sir Darcy Wentworth Thompson (1860-1948) states:-

"Of famous and fascinating numbers a mathematical friend writes to me: All the romance of continued fractions, linear recurrence relations,….lies in them, and they are a source of endless curiosity. How interesting it is to see them striving to attain the unattainable, the Golden ratio, for instance; and this is only one of hundred of such relations".

In answer to the afore mentioned question as to the genuine existence of the Golden ratio; the discovery of Incommensurability makes its existence all the more creditable, and a ratio is after all just the quotient of two quantities.

Euclid of Alexandria (325-265 BC) the Greek Geometer convoluted the situation further when he composed his text; The Elements, in which he proposes that geometry can be thought of in not just one dimension, but the Second dimension known as the "Plane" and also the Third Dimension known as "Space".

Ian Stewart (2008) describes the work of Euclid as "An examination of the logic of Spatial Relationships", "If a shape has certain properties, these may logically imply other properties".

At this stage we arrive back at the Theorem of Pythagoras (570- 495BC) whereby he states that, "In any right triangle the area of the square whose side is the Hypotenuse is equal to the sum of the areas of the squares whose sides are the two legs".

Euclid's work revealed that there are exactly five Regular Polyhedra or Platonic Solids in existence, that from a geometrical point of view they can actually be built and that their sides, faces and edges fit together perfectly with absolutely no divergence or error whatsoever.

Ian Stewart (2008) states that:-

"A solid is regular (or Platonic) if it is formed from identical faces, arranged in the same way at each vertex, with each face a Regular Polyhedron".

The Platonic Solids are listed as follows:-

1. The Tetrahedron, constructed of four Equilateral Triangles.
2. The Cube, constructed of six Squares.
3. The Octahedron, constructed of eight Equilateral Triangles.
4. The Dodecahedron, constructed of 12 Regular Pentagons.
5. The Icosahedron, constructed of 20 Equilateral Triangles.

The Platonic Solids have throughout history been inextricably linked to the Elements of Antiquity:- Earth, Water, Air and Fire with Quintessence which is also recognised by the term "The Fifth Element", forming the Icosahedron.

The Dodecahedron has Pentagonal faces and the five faces surrounding the vertex of the Icosahedron form a Pentagon the significance of this fact is due to the direct relationship between the Pentagon and what is termed as the "Extreme and Mean Ratio" also known as the "Golden Mean".

If we consider a line from A to B and then into the line position a point, C, if the ratio of AB – AC is the same as AC – BC then the complete line will proportionally correlate the larger section as the larger section does to the smaller section. A five pointed star built within a Pentagon has this very same relationship created within its edge lengths, these being the edge of the Pentagon and the two edges of the star, this is a precise geometrical description of the Golden Ratio. This ratio is equal to $1+\sqrt{5}\div 2$ and is Irrational, which in numerical terms has a value of approximately 1.618 the ancient Greek Geometers, by using Pentagonal Geometry proved that $1+\sqrt{5}\div 2$ and 1.618 are therefore Irrational.

The following thoughts are put forward to explain why the Golden Ratio would be significant in the dressing, shoeing and balancing of horses feet, but it must be for the reader to form their own opinion as to just how significant (Plate 17).

If we consider that the ground bearing surface of the foot is represented by the Hypotenuse (B-C) and the dorsal wall (A-B) and line from the hairline at mid-line dead centre to the last weight bearing point of the heels (A-C) are represented by the Legs of the right angle (A) triangle it would appear possible that a square built on the Hypotenuse (5) could represent ground reaction forces and the sum of the volume of the two squares built on the legs (3+4) could be said to represent load bearing forces. According to Pythagorean Theorem this means that the ground reaction (5) forces and the load bearing forces (3+4) would be of equal force, however if the right angle (A) triangle (Keplers Triangle and the Golden ratio) is not formed because of low heels (C) which in turn means that an acute dorsal wall angle (B) exists, thereby increasing the length of the Hypotenuse (B-C) and the angle at the hairline mid-line dead centre (A) and thus displacing the centres of the two triangles which should represent the Point of insertion of the Deep digital flexor tendon (X) and the Point of force (Y), then the volume of the applied force (3,4 + 5) would be unequal and this in turn would possibly increase the risk of morphological and pathological changes to the hoof and the structures proximal to it.

Albert Einstein (1874-1955) stated that:-

"As far as the laws of mathematics refer to reality, they are not certain; and as far as they are certain they do not refer to reality".

Pythagorean Theorem in relation to Forces Vectors

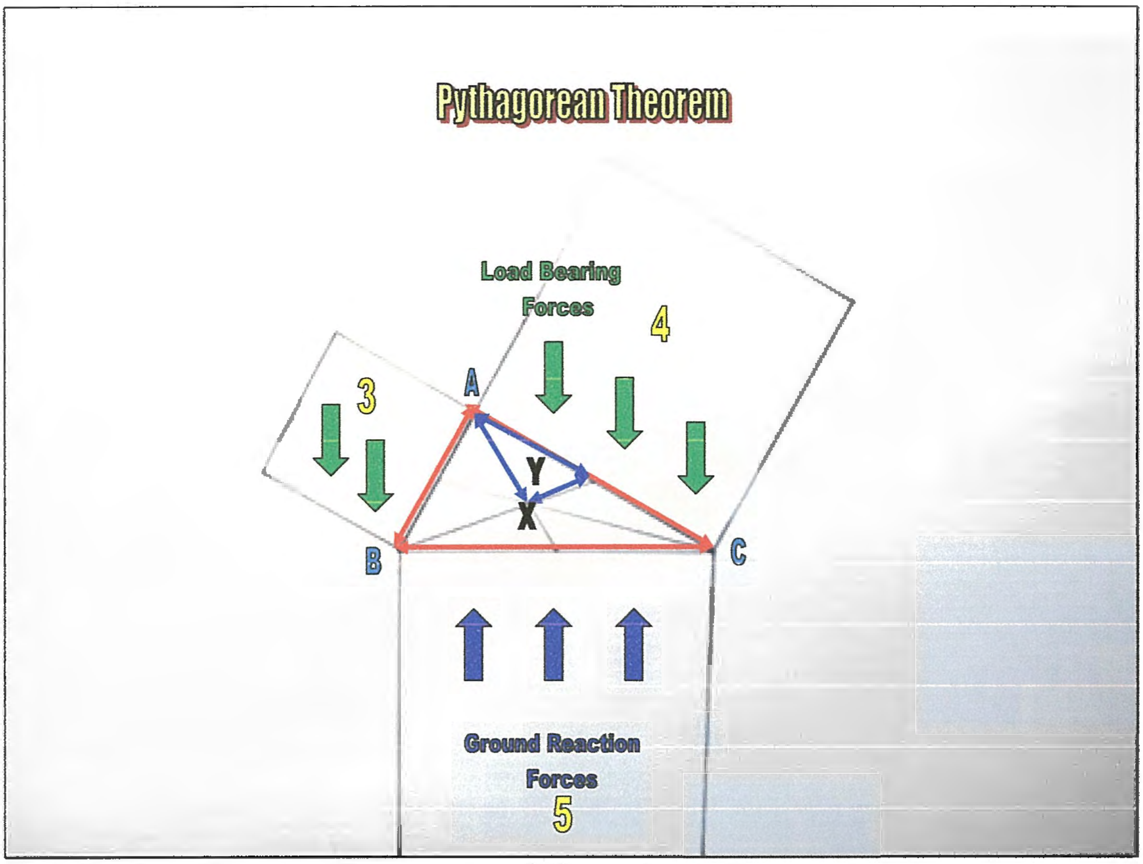

Plate 17:- Hoof Capsular Geometry in relation to Force Vectors.

Darcy Wentworth Thompson (1860-1948) claims that:-

"In an organism, great or small, it is not merely the nature of the motions of the living substance which we must interpret in terms of force (according to Kinetics), but also the conformation of the organism itself, whole permanence or equilibrium is explained by the interaction or balance of forces, as described in Statics"

Initially the possibility that the Golden Ratio might have a relationship in the search for the optimum method of dressing and trimming horses feet was a chance discovery made in the process of carrying out a previous study into optimum hoof balance, based on an attempt to align specific external reference points with specific internal anatomical structures. It was whilst examining measurements on digital images of saggital sections of the feet using the Ontrack software program on a Sony Vaio VGN-AR51E laptop, that after some months the author realised the possibility of a correlation between hoof capsular geometry on a saggital plane and the progressive edge lengths of a right angle triangle known to form the golden ratio and Keplers triangle.

None of the feet in this or previous studies were trimmed specifically to the approximate lengths to attempt to form the Golden Ratio. To whatever degree their may or may not be any form of correlation is purely a secondary factor, however convenient it could be perceived.

Due to Incommensurability, the correlation between the Golden Ratio and Optimum hoof balance is a far greater prospect, with a specific relationship to what is termed an Equiangular or Logarithmic Spiral Curve because a specific value cannot be obtained upon which to base a founding value.

William Butler Yeats (1865-1939) states that:-

"The very essence of genius, of whatever kind, is precision".

It would possibly have been a far greater discovery to have revealed definitively that no connection whatsoever exists between Optimum hoof balance and the Golden Ratio, instead of which we have a situation whereby all we can say at this stage, is that it is a very great probability.

Sophocles (495-405 B.C.) advocated that:-

"Numberless are the worlds wonders".

In the process of carrying out the practical aspect of the study a wealth of information was gathered, but possibly one of the greatest conclusions to be drawn from this study is that it is more likely to find the Golden Ratio in the foot of the well conformed limb, as visualised on presentation from the varying sources. Whether or not that relates to the individual animal can only be surmised as in most cases they were not seen by the author.

In some cases individual pairs differed greatly, whilst one foot formed the perfect Golden Ratio the other may have been far from it. Plate 18 shows the pair to the foot shown in Plate 5, and unlike Plate 5 it forms the perfect Golden ratio. As a result this horse displayed the Golden ratio in one foot but not the other (Table 1: 3A & 3B).

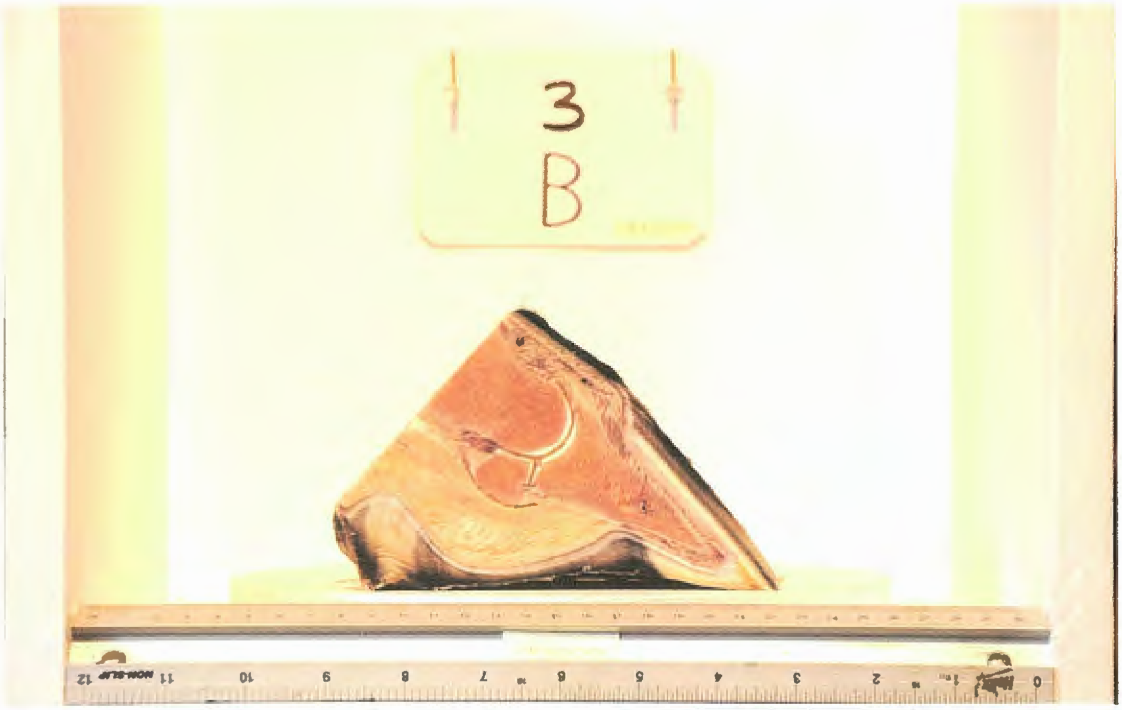

Plate 18:- Saggital section, showing the pair to the Lamanitic foot in plate 5.

In certain cases the individual hoof capsule geometry may be so poor as a result of the conformation above the foot that that it is impossible to imagine how an individual can stand let alone walk. Plate 19 shows a pair of fore limbs from a 21 year old Thoroughbred broodmare, neither foot related even remotely to the Golden Ratio, on a saggital plane (Table 1: 27A & 27B) she had an inch and a half difference in the length of her Third Metacarpals and yet she has bred a number of successful foals. Any number of conclusions can be drawn from this individual; perhaps this was why she was a broodmare? Could she have passed this trait on to her offspring? The questions are endless and yet she had a successful career regardless of her conformation. Perhaps this is the perfect example of an animal working within the confines of its individual conformation.

Plate 19:- Pair of forelimbs presenting with 1.5 inches difference in the lengths of the Third metacarpal.

In some cases the question has to be asked as to whether an individual had any chance of surviving in the first place. The foot shown in Plate 20 belonged to a 6 year old Warm blood gelding and had been shod 1 day prior to collection for this study.

Its shoeing protocol which was carried out by another Farrier was a direct result of Veterinary advice which has led to severe disproportionate capsular geometrics (Table 1: 44A & 44B).

If we consider the afore mentioned trimming protocol than the author had to remove three quarters of an inch from the heel of the foot to bring the last point of weight bearing to the widest point of the frog, nothing was removed from the solar surface or the Dorsal wall.

Plate 20:- Saggital section, revealing a misinterpretation of the required Geometrics. originating from excessive removal of the dorsal wall.

When we consider the fore limb conformation of this individual it becomes apparent that the hoof capsule geometry is as a direct result of poor conformation as shown in Plate 21 in conjunction with a misinterpretation of the farriery protocol required. The heels of both feet were equally in contact with the top edge of the rasp. this was done to give a perception of scale and the amount of limb deviation.

P.W.Balchin (2009) states that:-

"When we consider each individual limb it should be remembered that the physiological and morphological form of the hoof capsule will assume a representational mirrored image of the forces which have acted upon it and from this we can deduce the way in which these forces manifested themselves and how they influenced the structure and conformation of the hoof capsule as it developed".

Plate 21:- Forelimbs, belonging to the foot in plate 20.

The question has to be posed as to whether or not this animals demise was as a result of its conformation and, if the correct Geometric proportions had been imposed on its feet from an early age, would it be included in this study or perhaps it would form part of another study dealing with the hoof capsule geometry in the live animal?

Dr Johnson (1750) states that:-

"Mathematicians are well acquainted with the difference between pure sciences, which has to do only with ideas, and the application of its laws to the use of life, in which they are constrained to submit to the imperfections of matter and the influence of accident".

Study Limitations

This particular study was aimed at assessing the hoof capsular geometry in the fore limbs of the dead horse. The information regarding each individual is limited, it is however included in the appendices. A study dealing with the live animal would possibly be of greater benefit to all concerned with the study of Equine locomotor biomechanics particularly if it was carried out in conjunction with studying the performance level achieved in each individuals specific discipline.

The use of a different trimming protocol to that advocated by M.N.Caldwell et al (forge, April 2009) may produce some interesting results This could be achieved by obtaining the true Dorsal wall length and then trimming the bearing border to the appropriate proportions. Once this had been carried out an assessment could be made as to whether or not specific external reference points formed any form of correlation with specific internal anatomical structures.

Darcy Wentworth Thompson (1860-1948) states that:-

"Often it happens that our physical knowledge is inadequate to explain the mechanical working of the organism; the phenomena are superlatively complex, the procedure is involved and entangled, and the investigation has occupied but a few short lives of men".

Conclusion

The consideration that the Golden Ratio may have a role to play in the search for the optimum hoof balance is becoming a very real prospect and yet it should be treated with the greatest of respect and not deemed a hard and fast rule, more, instead a guideline; another tool which the practioner can refer to in conjunction with other previously established methods.

Mario Livio (2002) states that:-

"Pure mathematics usually refer to the type of mathematics that at least on the face of it has absolutely no direct relevance to the world outside the mind".

This is the first substantial piece of work that has been carried out with a view to assessing the Golden Ratio in relationship to Functional Hoof Capsular Geometry, hopefully in time others may follow and be able to expand on the subject, possibly by using this work as a basis.

It would appear that this study has uncovered many more questions than answers, however the answer to the question as to whether or not the Golden Ratio is significant to functional hoof capsular geometry must be based on the interpretation of the individual reading this piece of work.

Mario Livio (2002) states that:-

"Absence of Evidence is not Evidence of Absence".

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Appendices

Materials Check List:-

100 Cadaver Forelimbs, (50 Pairs).

Dewalt DW738 Electric Bandsaw.

Scandinova Chest Freezer.

Nikon D80 DSLR with a Nikon DX AF-S Nikkor 18-135mm Lens.

Nikon ML-L3 Infrared Remote Control.

Giottos MTL9351B Tripod.

Sony Vaio VGN-AR51E Laptop Computer.

Ontrack Software.

Calibrated Photographic Box.

Powerfix Digital Vernier Calliper.

J.H.Forge Calibrated Dividers.

Clean Boiled Water.

Latex Gloves.

Marigold Gloves.

Hibi Scrub.

Dryline Marker.

Brass Ruler.

Tooth Brush.

Clean Cotton Towels.

Leyland White Matt Paint.

Soft Nylon Brush.

Freezer Bags.

Body Bags.

Materials Check List continued:-

Halogen Light.

Gibbons Chaps.

G.E. Snips.

Heller Legend Rasps.

Paul Mitchell Loop Knife.

Paul Mitchell Knife Sharpener.

Frank Ringel Straight Knife.

Buck Folding Kalinga Pro Hunter Knife.

Whitby Filleting Knife.

Soft Wire Brush.

Adhesive Labels.

Burco Boiler.

Basic Information that could be gleaned about some Individuals:-

3A/B, 16 year old Thoroughbred Brood mare, Laminitis in foot B, Concave Heart Bar had been fitted.

4A/B, 21 year old Thoroughbred X mare, (Lady Warren). Authors client for 12 years.

5A/B, Advanced Show Hunter, 7 year old, Grey, Rockstar.

9A/B, Thoroughbred mare, 10 months old.

11A/B, Laminitic Pony.

12A/B, Thoroughbred Yearling.

13A/B, Aged Thoroughbred gelding, Concave Side bone shoes had been fitted to both feet. Foot B presented with a severe medio/lateral imbalance.

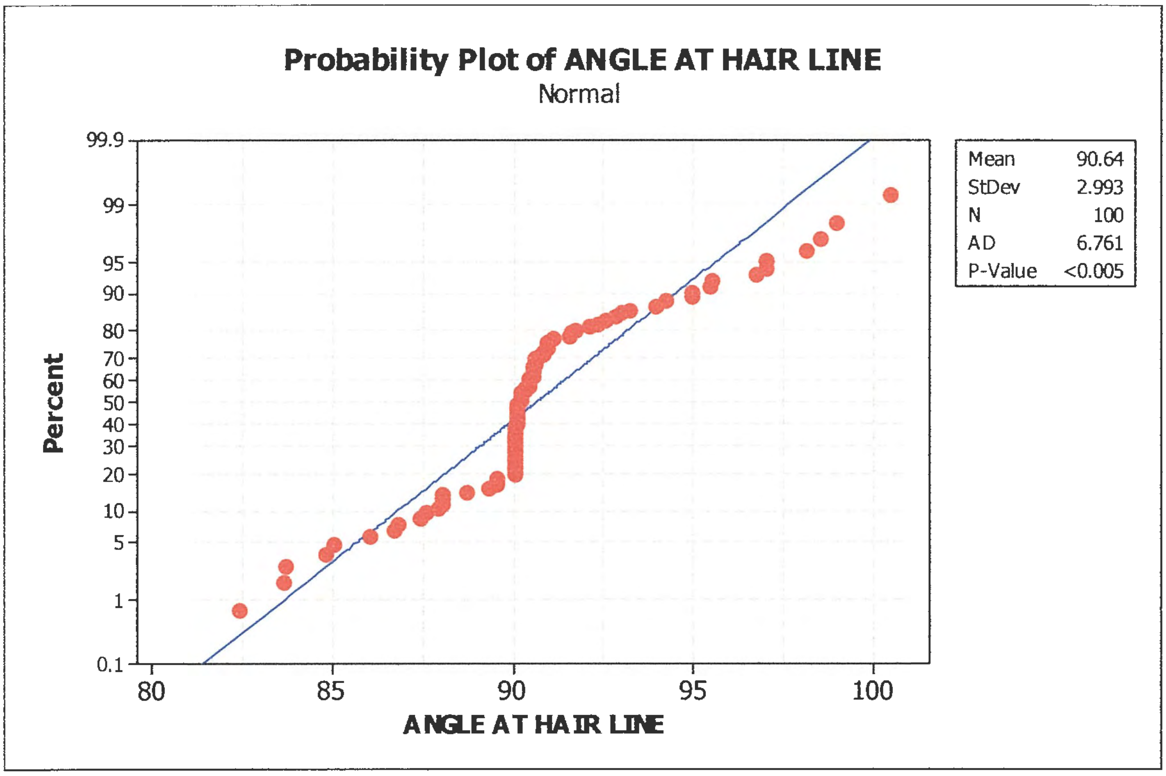
17A/B, Irish Hunter, gelding 10 year old, broke limb B mid 3rd Metacarpal in a Badgers Earth.

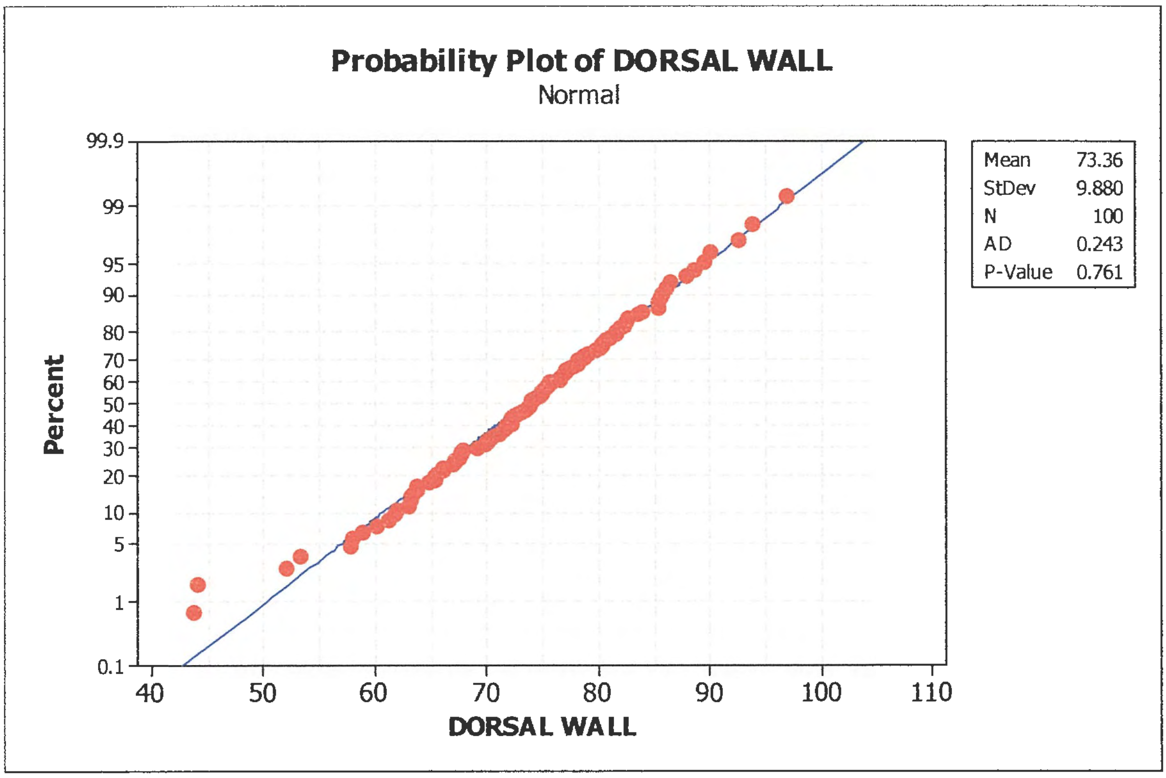
26A/B, Aged Thoroughbred broodmare.

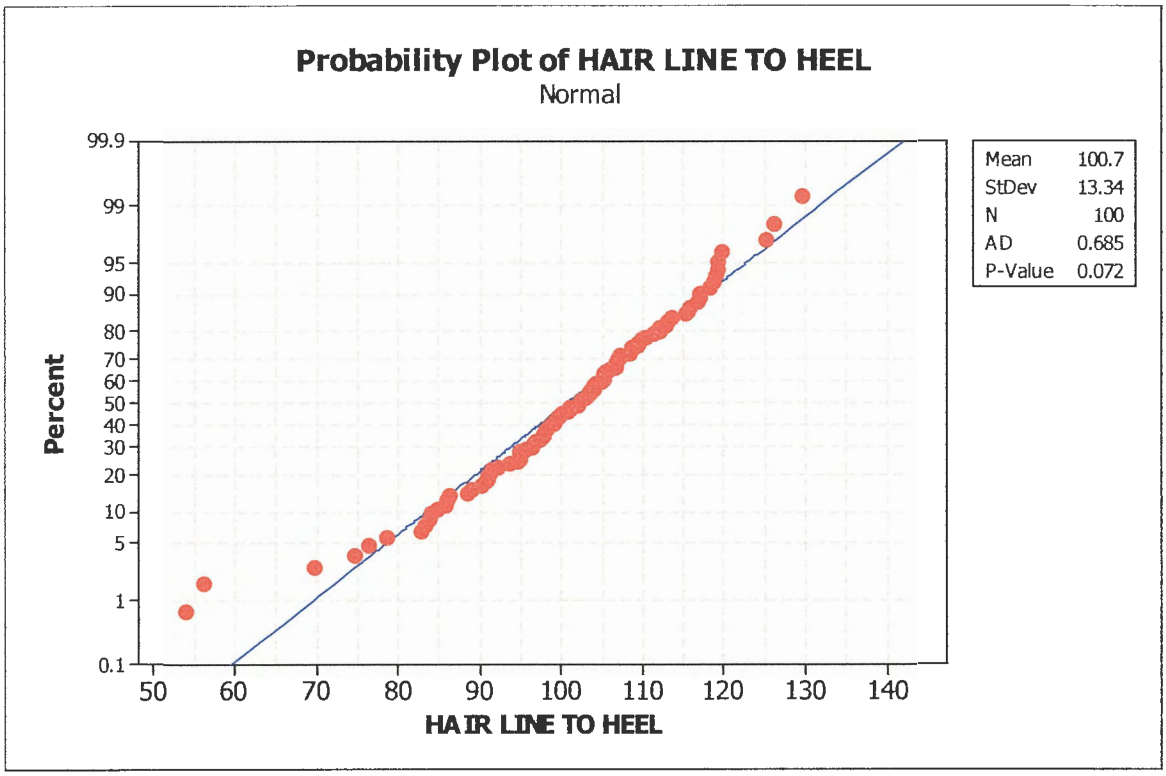
27A/B, Aged Thoroughbred broodmare, 3rd Metacarpal A 1.5 inches shorter than 3rd Metacarpal B.

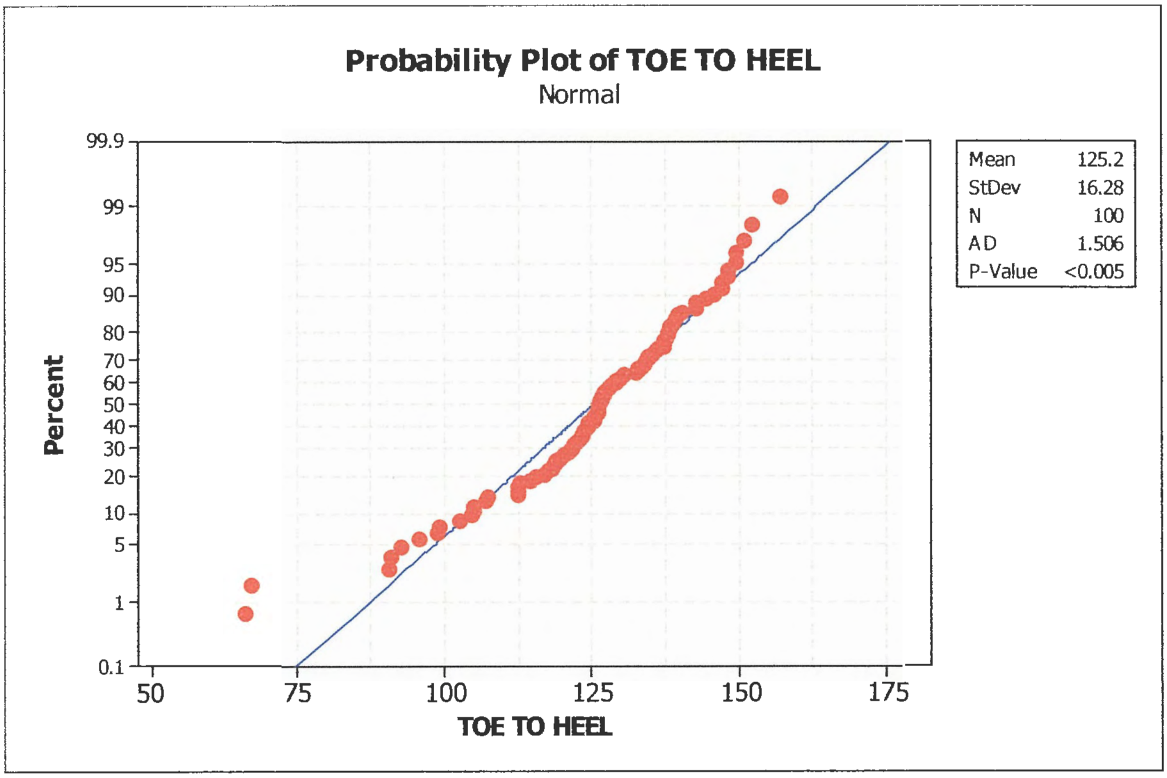
28A/B, Aged Thoroughbred gelding, Limb B presents with severe Carpo-Metacarpal Joint infection, Joint showed signs of advanced degeneration and smelt rancid.

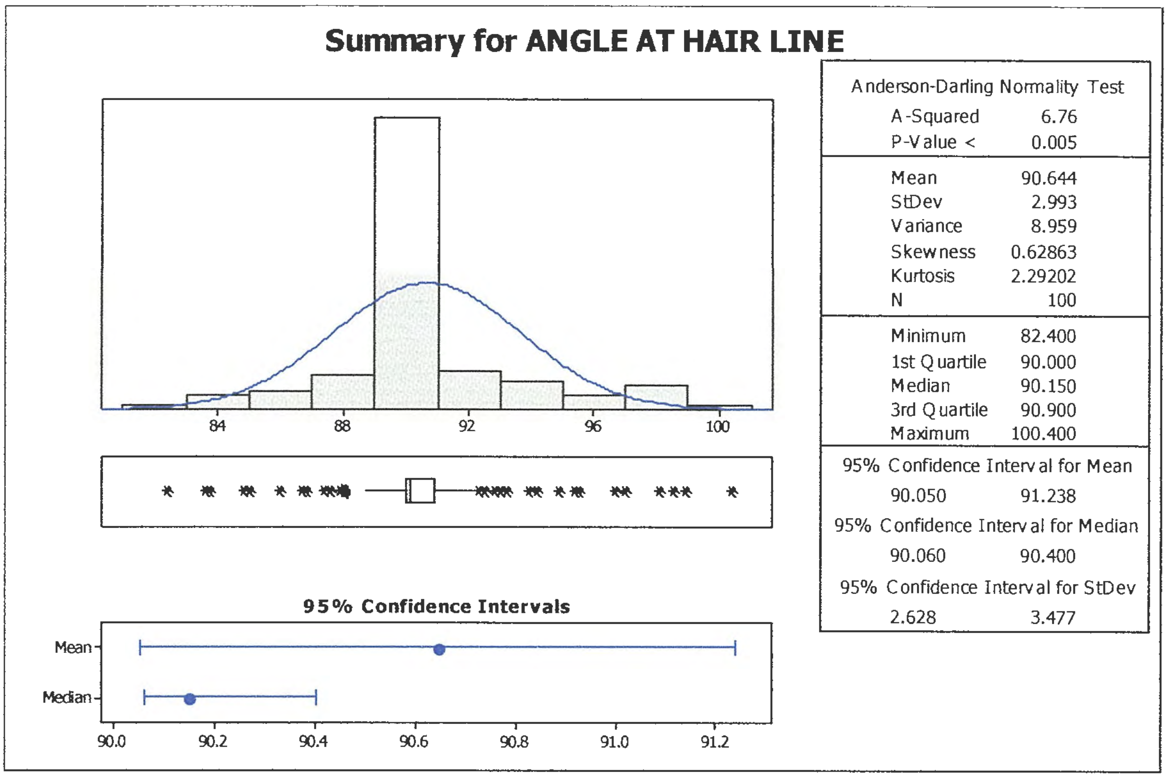
29A/B, 2 year old Thoroughbred mare. Gorged itself to death in grain barn.

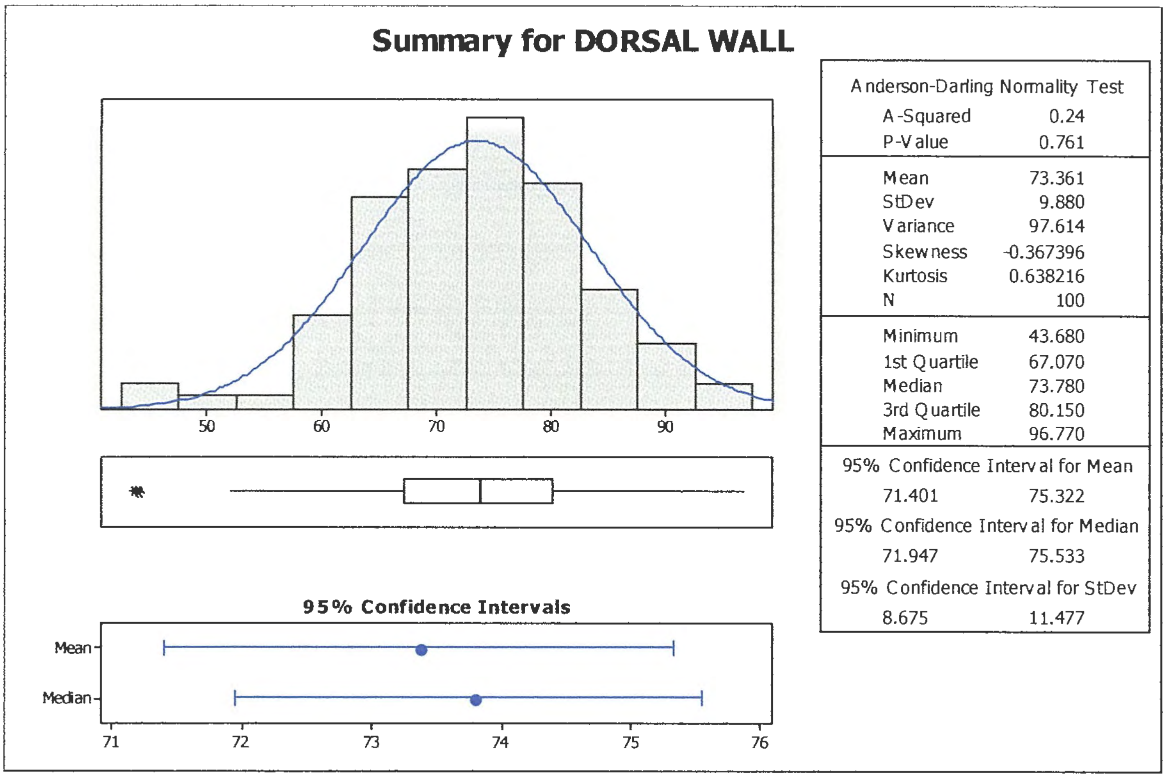

Probability Plot of ANGLE AT HAIR LINE
Normal
Percent
99.9
99
95
90
80
70
60
50
40
30
20
10
5
1
0.1
80
85
90
95
100
ANGLE AT HAIR LINE
Mean 90.64
StDev 2.993
N 100
AD 6.761
P-Value <0.005

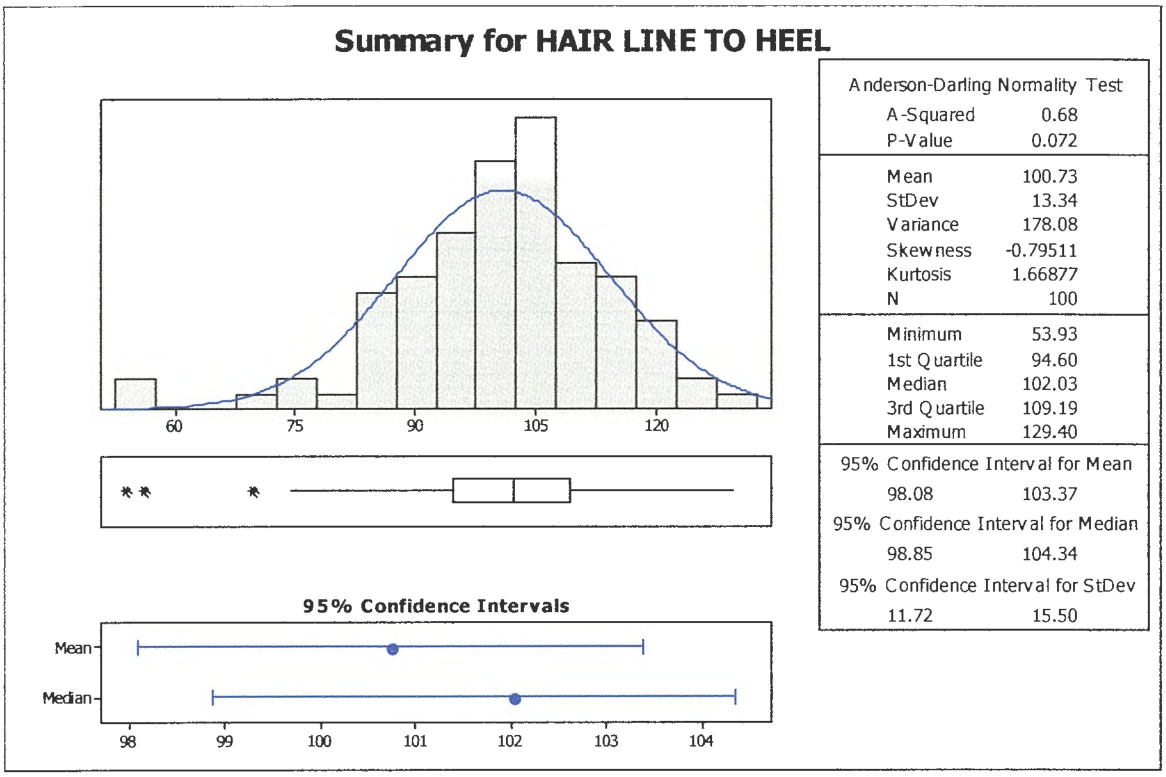

Probability Plot of DORSAL WALL
Normal
Percent
99.9
99
95
90
80
70
60
50
40
30
20
10
5
1
0.1
40
50
60
70
80
90
100
110
DORSAL WALL
Mean 73.36
StDev 9.880
N 100
AD 0.243
P-Value 0.761

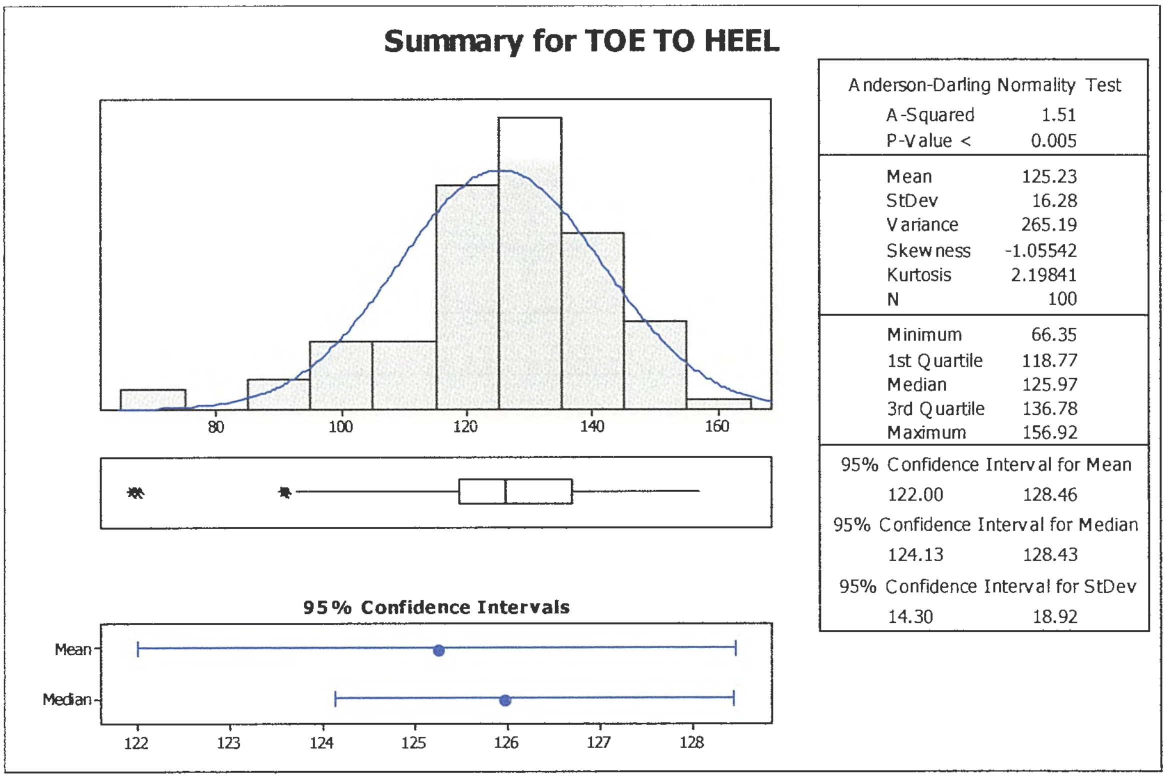

Probability Plot of HAIR LINE TO HEEL
Normal
Percent
99.9
99
95
90
80
70
60
50
40
30
20
10
5
1
0.1
50
60
70
80
90
100
110
120
130
140
HAIR LINE TO HEEL
Mean 100.7
StDev 13.34
N 100
AD 0.685
P-Value 0.072

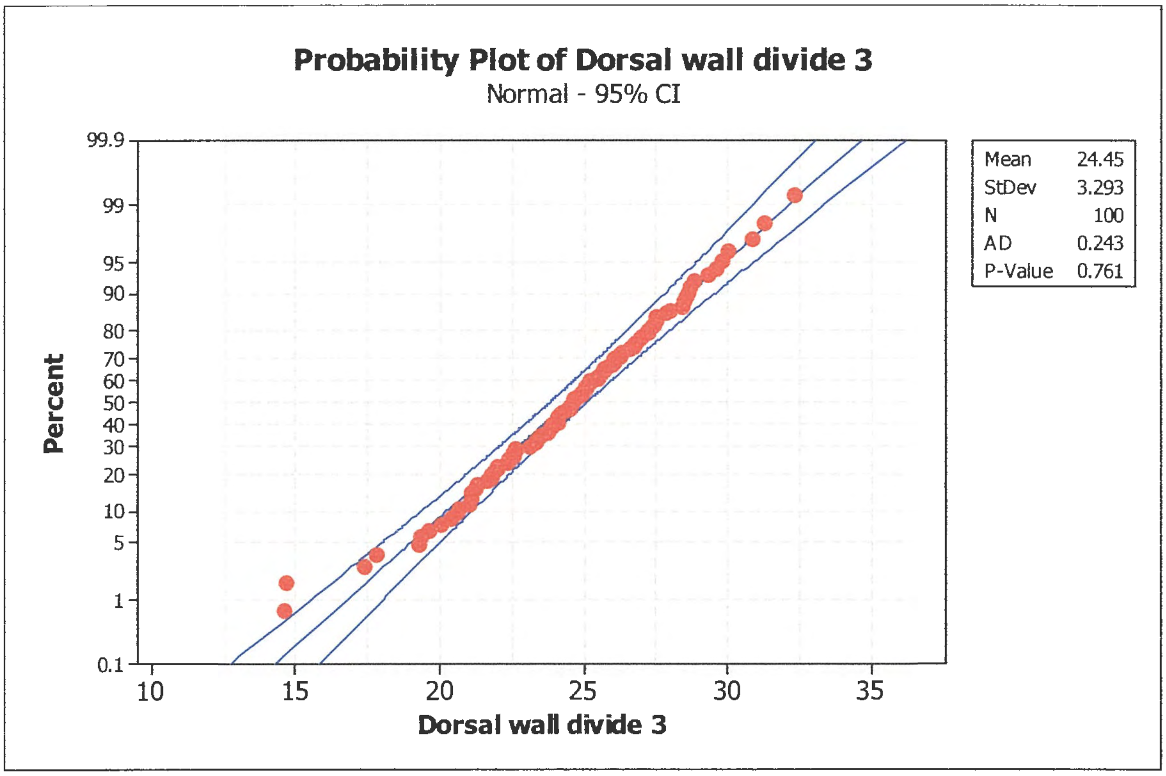

Probability Plot of TOE TO HEEL
Normal
Percent
99.9
99
95
90
80
70
60
50
40
30
20
10
5
1
0.1
50
75
100
125
150
175
TOE TO HEEL
Mean 125.2
StDev 16.28
N 100
AD 1.506
P-Value <0.005

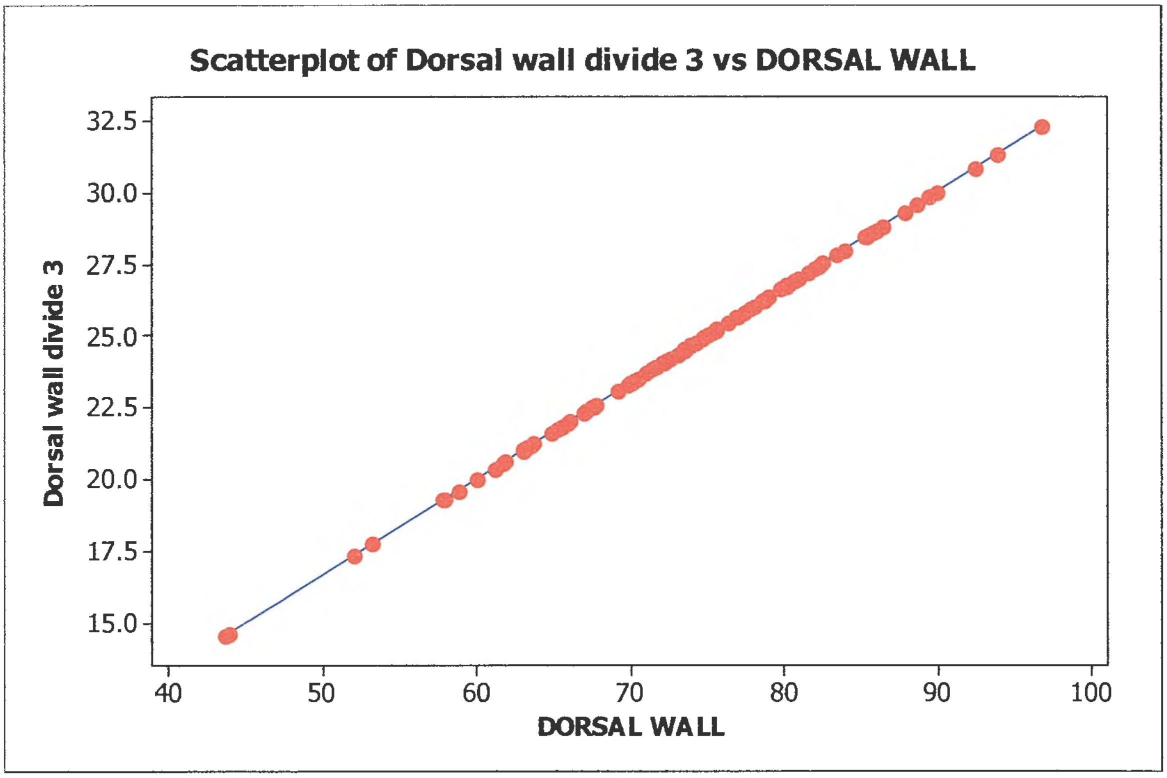

Summary for ANGLE AT HAIR LINE
Anderson-Darling Normality Test
A-Squared 6.76
P-Value < 0.005
Mean 90.644
StDev 2.993
Variance 8.959
Skewness 0.62863
Kurtosis 2.29202
N 100
Minimum 82.400
1st Quartile 90.000
Median 90.150
3rd Quartile 90.900
Maximum 100.400
95% Confidence Interval for Mean
90.050 91.238
95% Confidence Interval for Median
90.060 90.400
95% Confidence Interval for StDev
2.628 3.477
84
88
92
96
100
95% Confidence Intervals
Mean
Median
90.0
90.2
90.4
90.6
90.8
91.0
91.2

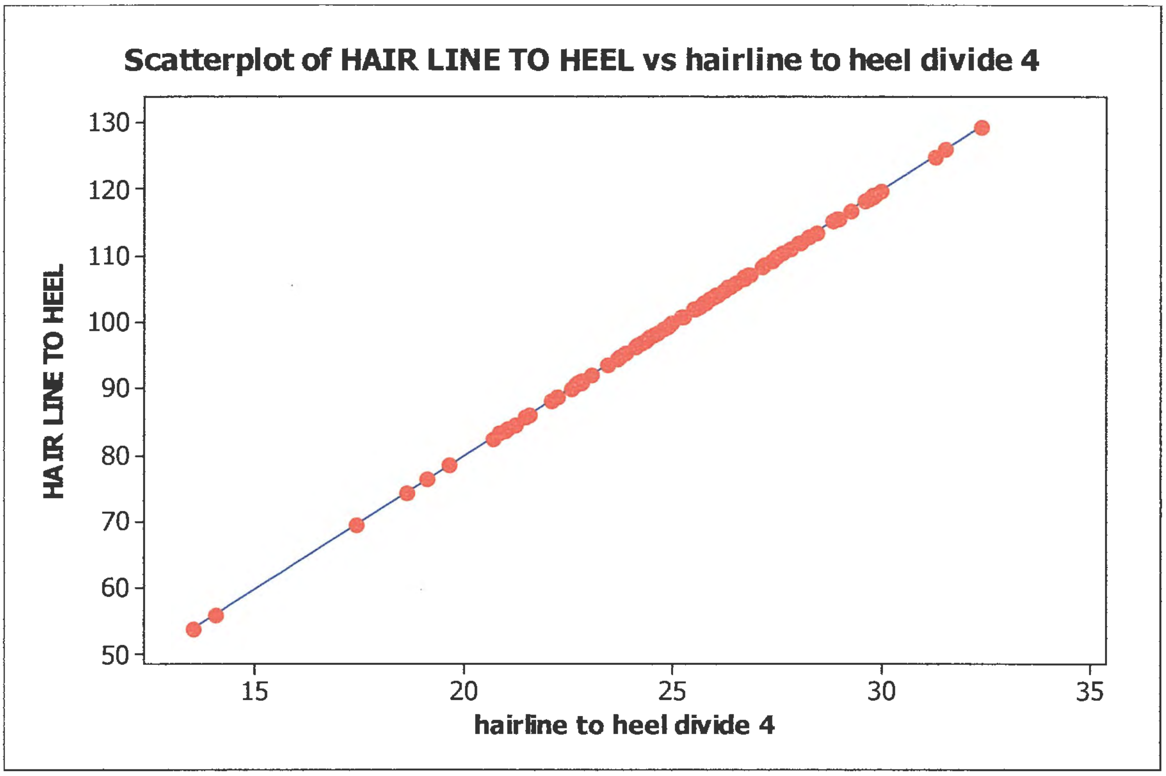

Summary for DORSAL WALL
Anderson-Darling Normality Test
A-Squared 0.24
P-Value 0.761
Mean 73.361
StDev 9.880
Variance 97.614
Skewness -0.367396
Kurtosis 0.638216
N 100
Minimum 43.680
1st Quartile 67.070
Median 73.780
3rd Quartile 80.150
Maximum 96.770
95% Confidence Interval for Mean
71.401 75.322
95% Confidence Interval for Median
71.947 75.533
95% Confidence Interval for StDev
8.675 11.477
50
60
70
80
90
95% Confidence Intervals
Mean
Median
71
72
73
74
75
76


Summary for HAIR LINE TO HEEL
Anderson-Darling Normality Test
A-Squared 0.68
P-Value 0.072
Mean 100.73
StDev 13.34
Variance 178.08
Skewness -0.79511
Kurtosis 1.66877
N 100
Minimum 53.93
1st Quartile 94.60
Median 102.03
3rd Quartile 109.19
Maximum 129.40
95% Confidence Interval for Mean
98.08 103.37
95% Confidence Interval for Median
98.85 104.34
95% Confidence Interval for StDev
11.72 15.50
60 75 90 105 120
95% Confidence Intervals
Mean
Median
98 99 100 101 102 103 104

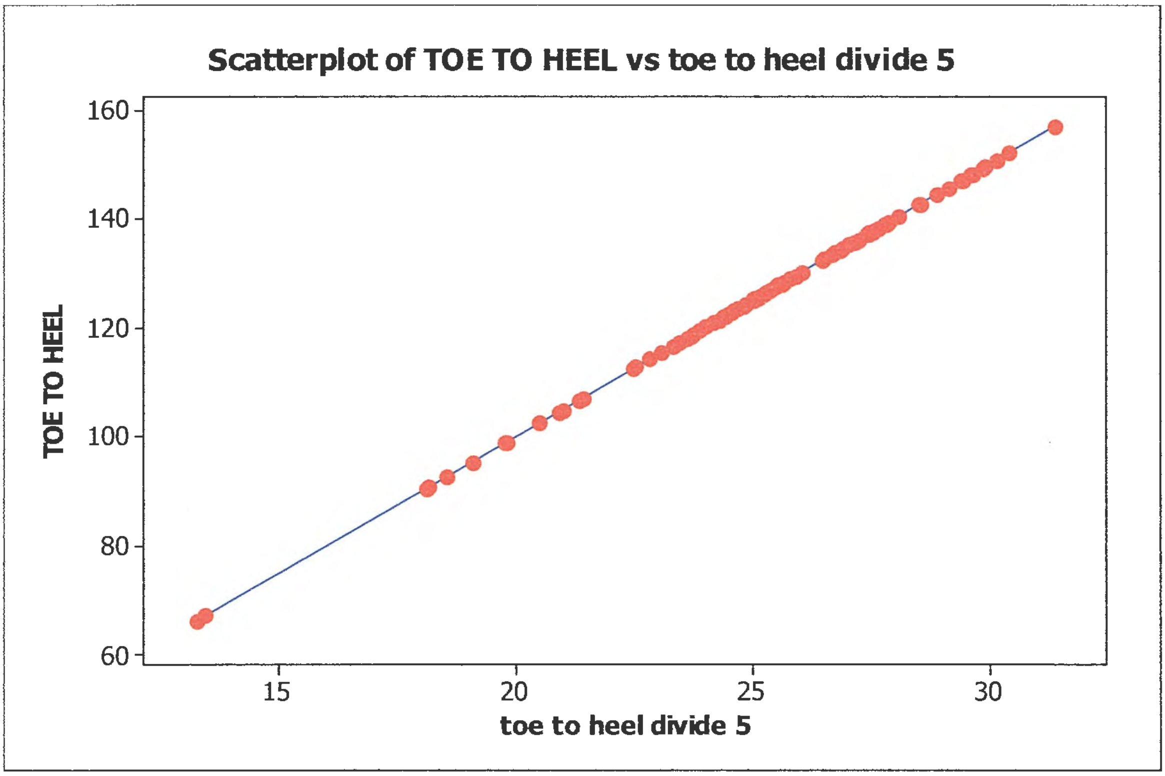

Summary for TOE TO HEEL
Anderson-Darling Normality Test
A-Squared 1.51
P-Value < 0.005
Mean 125.23
StDev 16.28
Variance 265.19
Skewness -1.05542
Kurtosis 2.19841
N 100
Minimum 66.35
1st Quartile 118.77
Median 125.97
3rd Quartile 136.78
Maximum 156.92
95% Confidence Interval for Mean
122.00 128.46
95% Confidence Interval for Median
124.13 128.43
95% Confidence Interval for StDev
14.30 18.92
80 100 120 140 160
95% Confidence Intervals
Mean
Median
122 123 124 125 126 127 128


Probability Plot of Dorsal wall divide 3
Normal - 95% CI
Mean 24.45
StDev 3.293
N 100
AD 0.243
P-Value 0.761
Percent
99.9
99
95
90
80
70
60
50
40
30
20
10
5
1
0.1
10
15
20
25
30
35
Dorsal wall divide 3

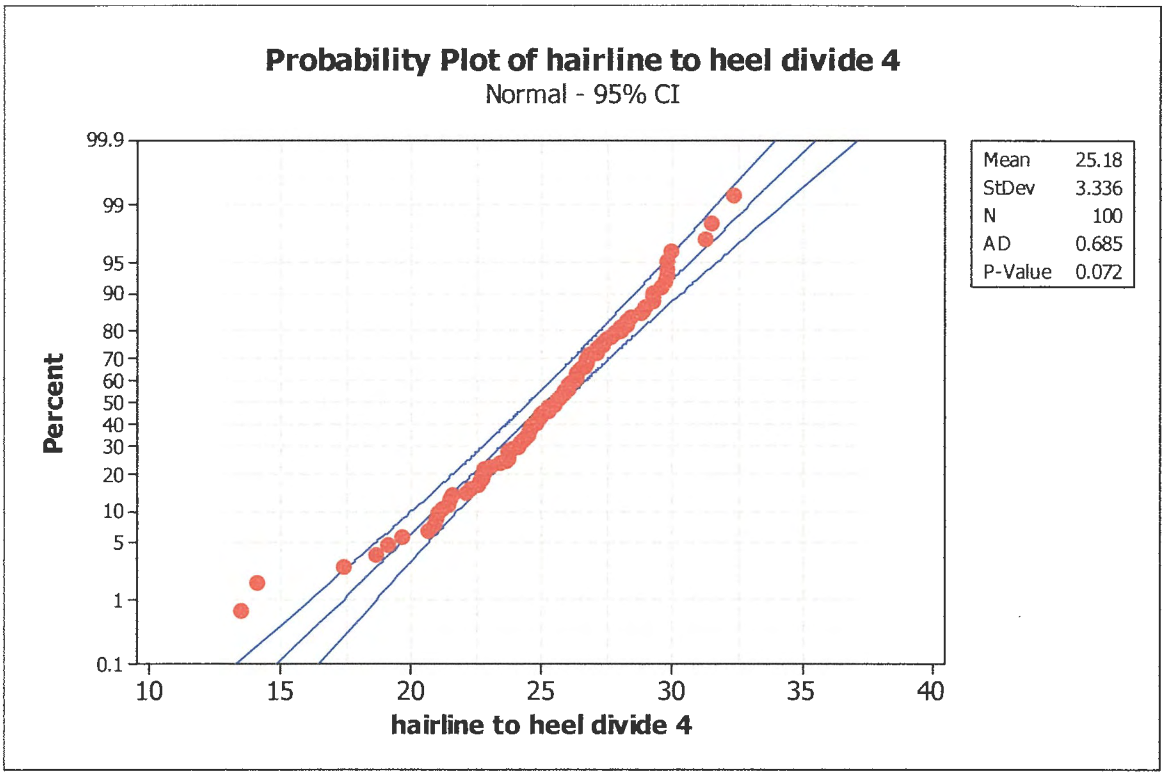

Scatterplot of Dorsal wall divide 3 vs DORSAL WALL
Dorsal wall divide 3
32.5
30.0
27.5
25.0
22.5
20.0
17.5
15.0
40
50
60
70
80
90
100
DORSAL WALL

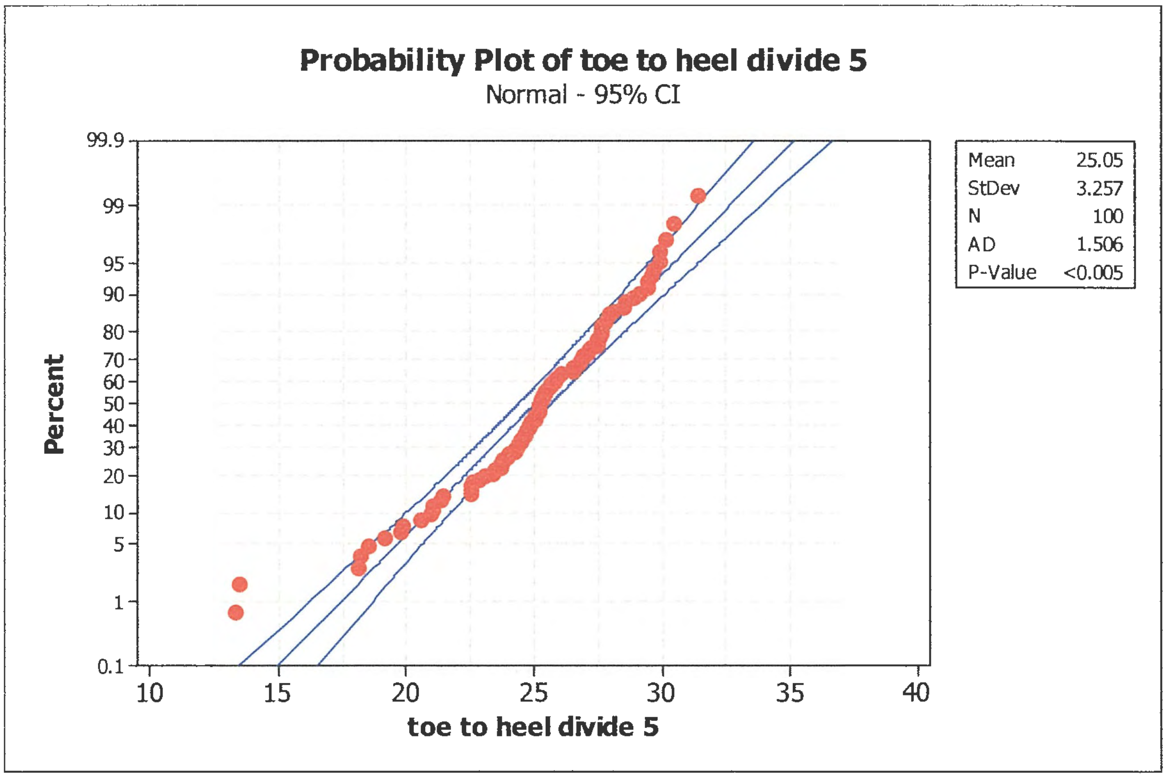


Scatterplot of HAIR LINE TO HEEL vs hairline to heel divide 4
HAIR LINE TO HEEL
130
120
110
100
90
80
70
60
50
15
20
25
30
35
hairline to heel divide 4



Scatterplot of TOE TO HEEL vs toe to heel divide 5
TOE TO HEEL
160
140
120
100
80
60
15
20
25
30
toe to heel divide 5





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